A Short and Sweet Refutation of

Gödel's Theorem

by

Ardeshir Mehta

(This version finalised on Monday, August 6, 2001)

Argument

The Gödel-number of every formula of "the system **P**" which contains a "number-sign" (or "numeral") must be greater than the numerical value of that number-sign itself.

For instance, if any formula of the system **P** contains the number-sign for the number 17, then its Gödel-number cannot possibly be equal to or less than 17.

This is because by definition, every such formula *itself* contains a number-sign, and this number-sign *itself* consists of a number of basic signs of the system **P**, this number of basic signs being one more than the natural number *of* which it is the number-sign.

Therefore the Gödel-number of that number-sign *alone* must be greater than the numerical value of that number-sign itself. (This does not even take into consideration the rest of the formula!)

Example

Let ***r*** be the Gödel-number of what Gödel calls a "class-sign" (*i.e.,* a well-formed formula with a single free variable); and let that free variable be ***q***. Then ***r*** is, essentially:

some-sequence-of-basic-signs_q_some-other-sequence-of-basic-signs

If we now define the function **Sb (*r q* | Z(*x*))** as being the propositional formula obtained by the substitution, in the class-sign of which the Gödel-number is ***r,*** of the only free variable in it, ***q,*** by number-sign for the number ***x,*** then the sequence **Sb (*r q* | Z(*x*))** becomes:

some-sequence-of-basic-signs_f f f ... s0_some-other-sequence-of-basic-signs

… where ***f*** means "the successor of". Notice that there must be ***x*+1** basic signs in the sequence ***f f f*... *f*0**.

However, by the system called Gödel-numbering, the basic sign ***f*** is given a Gödel-number which is a non-zero natural number — and which is always, as a result, either 1 or greater than 1.

Thus the Gödel-number of the propositional formula **Sb (*r q* | Z(*x*))** cannot possibly be equal to or less than ***x***, since the number of basic signs in the sequence ***fff... f*0**, as noted above, is ***x*+1**; and in addition there are the other basic signs in the formula, whose Gödel-numbers are also 1 or greater than 1, and which therefore contribute towards increasing the value of the Gödel-number of the formula **Sb (*r q* | Z(*x*))**.

In other words, the Gödel-number of *any* propositional formula containing a number-sign — *e.g.,* a formula resulting from the substitution of the only free variable in a class-sign by a number-sign — must *always* be greater than the numerical value of that number-sign itself. Both its Gödel-number *and* the numerical value of the number-sign contained in it cannot possibly be equal to one another.

THE SO-CALLED "UNDECIDABLE FORMULA"

Now let us assume, as our hypothesis, that there *is* in fact an undecidable propositional formula in the system **P**, and that ***g*** is its Gödel-number. By the above argument, the numerical value of ***g*** must be *greater* than the numerical value of the number-sign contained in it.

However, according to Gödel's argument, the purported undecidable formula refers to *itself;* and thus ***g*** must *also* be the numerical value of the number-sign contained *in* the undecidable formula.

Therefore ***g*** *cannot* be the Gödel-number of any undecidable formula. Indeed it cannot be the Gödel-number of *any* propositional formula which refers to itself! If it were, it would contain a number-sign whose numerical value would be equal to ***g***. And in that case the numerical value of that number-sign would have to be both exactly equal to *and* greater than ***g*** — which is logically impossible.

Since the implications of our hypothesis lead to an impossibility, the hypothesis itself cannot have been correct; and as a result, there can be no number ***g*** which is the Gödel-number of any "undecidable formula". Indeed there can be no number ***g*** which is the Gödel-number of *any* propositional formula that refers to itself.

(*By the way:* It may not be argued that the so-called "undecidable formula" need not *itself* contain its own Gödel-number, but may refer to itself *indirectly,* by saying, in effect, "That formula which is obtained by substituting the free variable in formula number so-and-so by the number-sign of its own Gödel-number is not provable". As Gödel himself writes — in footnote No. 20 and Definition No. 31of his 1931 paper entitled "On Formally Undecidable Propositions of Principia Mathematica and Related Systems", wherein he tries to prove his celebrated Theorem — such a formula , containing as it must the sign "Subst" or "Sb" (standing for the operation of substituting a variable in a formula with a number) belongs to *meta*mathematics, not to *mathematics.* Thus if Gödel *can* in fact prove that such an undecidable formula exists, he can only prove thereby that mathematics by itself cannot decide a *meta*mathematical formula … to which we should retort, as any school-boy justifiably might: "*Big deal!*")

COMMENTS

If you have any comments, please e-mail me.

ACKNOWLEDGEMENT

I got the idea for this argument after reading an article entitled *On Gödel's Formula* by Jailton C. Ferreira, which can be downloaded in .pdf (*Adobe Acrobat*) format from:

<http://arXiv.org/abs/math/0104025>

POSTSCRIPT

Of course Gödel-numbers *themselves* belong to metamathematics, and not to mathematics, and may not validly be used in any mathematical formula. Any formula in which a Gödel-number appears must belong to metamathematics, not to mathematics; and thus if Gödel *can* actually prove that there *is* such a formula and that it *is* indeed undecidable, all he can possibly prove thereby is that it is his *meta*mathematics that is incomplete ... leaving mathematics *itself* as complete as ever it was!

POST-POSTSCRIPT

One can hardly argue that mathematics and metamathematics are essentially the same thing: for if they were, it should be possible to derive *all* of metamathematics from the axioms of mathematics *alone* (such as the Peano axioms, or the axioms of Zermelo and Fraenkel, later extended by John von Neumann). But no one, not even Gödel, has ever been able to lay claim to having performed such a superhuman feat.

POST-POST-POSTSCRIPT

There is a much fuller, but still comprehensible, account of the above ideas in my book *Critique of Gödel's 1931 Paper Entitled "On Formally Undecidable Propositions of Principia Mathematica and Related Systems",* which I wrote last year in collaboration with Ferdinand Romero, and which is available for download in .pdf format from my Home Page.