LIAR PARADOX IN A LOGIC WITH AN UNBOUNDED NUMBER OF TRUTH VALUES ("FUZZY LOGIC")

TO SHOW HOW THE "LIAR PARADOX" CAN BE FORMULATED IN ANY LOGIC WHICH ADMITS OF A FINITE NUMBER OF TRUTH-VALUES,
BUT NOT IN A LOGIC WHICH ADMITS OF AN UNBOUNDED NUMBER OF TRUTH-VALUES

by
Ardeshir Mehta

Friday, May 26, 2000

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1. Assume Two-valued Logic:

Let any proposition **p** admit of two (and *only* two) truth-values[i]:

1. 100% (or 1.0) true, or

2. 0% (or 0.0) true (*i.e.,* false).

Semantically, the meaning of this is that the membership of the proposition **p** in the set of all propositions that are true is either 1.0 or 0.0 — in other words, that **p** *is* a member of the set of all propositions that are true, or is *not* a member of that set. Absolutely *no* other possibilities are allowed.

In the notation of symbolic logic, this would read:

$$(\mathbf{q} \vee \sim\mathbf{q})$$

or …

$$(\mathbf{q} \vee \sim\mathbf{q}) \wedge \sim(\mathbf{q} \wedge \sim\mathbf{q})$$

[Here, the symbols "∨" and "∧" stand for the "or" and "and" operators, respectively, and the symbol "~" for the "not" operator.]

The above are, of course, the axioms of two-valued logic.

Now, using the notation of Prof. Karlis Podnieks, Dr. Math., University of Latvia, Institute of Mathematics and Computer Sciences: e-mail podnieks@cclu.lv — see also <http://www.ltn.lv/~podnieks/gt5.html#BM5_1>) let following proposition **q** be asserted:

q: q is false

… or, in the notation of Dr Dale Myers of the University of Hawaii, Dept. of Mathematics: dale@math.hawaii.edu, writing in the "Math Insight Project" (see <http://www.math.hawaii.edu/~dale/godel/godel.html>):

q iff q is false

… or, in the notation of symbolic logic,

q ≡ ~q

***Notes*:**

[1] For the purposes of the following argument the symbol "**:**" means "such that";

[2] the term "**iff**" means "if and only if", and for the purposes of the following argument may be considered semantically equivalent to"**:**" meaning "such that";

[3] the symbol "≡" means "is materially equivalent to", and for the purposes of the following argument may be considered semantically equivalent to "**iff**";

[2] Tarski's *Self-Reference Lemma* — which for the purposes of the following argument may be accepted as being both true and satisfactorily proven — states that in adequate mathematical theories, such equations as

q: q is false

… always have solutions.

Now this proposition

q ≡ ~q

… is the same as saying:

q: q is false (*i.e.*, q is 0.0 true)

or …

q iff q is false (*i.e.*, q is 0.0 true)

or …

~(q ≡ ~q)

And *this* is the same as saying:

q: q is false (*i.e.*, q is 0.0 true) but q is not 1.0 true

or …

q iff q is false (*i.e.*, q is 0.0 true) but q is not 1.0 true

or …

$$\mathbf{q} \equiv \sim\mathbf{q} \wedge \sim\mathbf{q}$$

(*Note:* Since there is no symbolic notation for the natural language term "but", for the purposes of the following argument it may be considered semantically equivalent to the logical operator "and", namely "∧".)

In consequence of the above statements, if two-valued logic is assumed, a paradox results, since:

(1) If **q** is (100%, or 1.0) true, then **q** is *not* 0.0 true. A contradiction results.

(2) If **q** is 0.0 true, then it asserts an absolute truth (*i.e.,* it can be inferred that **q** *must* be 1.0 true). Again, a contradiction results.

Since there are *no* other possibilities, a paradox results due to the above two contradictions.

Indeed for this reason, in two-valued logic the term

$$\mathbf{q} \equiv \sim\mathbf{q}$$

… is not allowed.[ii]

2. Assume Three-valued Logic:

Let any proposition **p** admit of three (and *only* three) truth-values:

(3) 100% (or 1.0) true, or

(4) 0% (or 0.0) true (*i.e.,* false) or

(5) 50% (or 0.5 or 1/2) true (or this "indeterminate".)

Semantically, the meaning of this is that the membership of the proposition **p** in the set of all propositions that are true is either 1.0, or 0.0, or both 1.0 and 0.0 to the degree of 50% membership in each — or in other words, that **p** *is* a member of the set of all propositions that are true, or is *not* a member of that set, or is *both* a member and *not* a member of that set. No other possibilities besides these three are allowed.

In symbols, these axioms can be expressed as:

$$(\mathbf{q} \vee \sim\mathbf{q} \vee \boldsymbol{i}\mathbf{q})$$

… where the symbol "***i***" stands for "indeterminate".

Now if we assert the following proposition **q** (and for the sake of brevity we shall henceforth dispense with the "**iff**" notation):

q: q is false

or …

$$\mathbf{q} \equiv \sim\mathbf{q}$$

… then this does *not* result in a paradox, for if **q** is indeterminate, or has a truth-value of (1/2) or 0.5, then **q** can be *both* false *and* true. Thus assuming that **q** is indeterminate, then the following relation holds:

q: q is both true and false

or …

$$\mathbf{q} \equiv \boldsymbol{i}\mathbf{q} \equiv (\mathbf{q} \vee \sim\mathbf{q})$$

… which is to say,

q: q is indeterminate

or, in another notation,

$$\mathbf{q} \equiv \mathbf{0.5q}$$

or, in yet another notation,

$$\mathbf{q} \equiv \mathbf{(1/2)q}$$

Under such circumstances, the proposition

q: q is false

or …

$$\mathbf{q} \equiv \sim\mathbf{q}$$

is itself only half true (or indeterminate), which is to say,

i(q ≡ ~q)

As a result of which:

iq ≡ iq

[Note that under three-valued logic, **i(~q) ≡ iq].**

As a result, no contradiction ensues, and thus no paradox results.

BUT if the following proposition is asserted:

q: q is false — (*i.e.,* q is 0.0 true) — or q is 0.5 true (*i.e.,* q is indeterminate).

… which is the same as saying:

q: q is false — (*i.e.,* q is 0.0 true) — or q is 0.5 true (*i.e.,* q is indeterminate), but q is not 1.0 true.

or …

q ≡ ~q ∨ iq

or …

q ≡ (~q ∨ iq) ∧ ~q

In this case, a paradox *does* result, since, upon opening out the term above, we get two terms separated by the ∨ ("or") operator:

(q ≡ ~q ∧ ~q) ∨ (q ≡ iq ∧ ~q)

… both of which terms (*i.e.,* the ones to the right and to the left of the ∨ operator) are disallowed by the axioms and rules of inference of the above-defined three-valued logic.

Or, in plain language:

(6) If **q** is (100%, or 1.0) true, then **q** is neither 0.0 true nor 0.5 true. A contradiction results.

(7) If **q** is 0.0 true, then by inference it asserts an absolute truth (*i.e.,* **q** is 1.0 true). Again, a contradiction results.

(8) If **q** is 0.5 true, then too by inference it asserts an absolute truth (*i.e.,* **q** is 1.0 true). Once again, a contradiction results.

Since there are *no* other possibilities, a paradox results due to the above three contradictions.

The same sort of argument can be extended to four-valued logic, five-valued logic, six-valued logic, … ***n***-valued logic (where ***n*** is any finite integer greater than two).

For example,

3. Assume Five-Valued Logic

Let any proposition **p** admit of five (and *only* five) truth-values:

(1) A truth-value of 1.0 (*i.e.*, totally or absolutely true), or

(2) A truth-vale of 0.0 (*i.e.*, totally or absolutely false) or

(3) 0.*a* true (where *a* is any finite integer greater than zero),

(4) 0.[*a*+*b*] true (where *b* is likewise any finite integer greater than zero), or

(5) 0.[*a*+*b*+*c*] true (where *c* is, again likewise, any finite integer greater than zero).

Now assert the following proposition **q**:

> **q: q is false — (*i.e.*, q is 0.0 true) — or q is 0.*a* true, or q is 0.[*a*+*b*] true or q is 0.[*a*+*b*+*c*] true.**

This is the same as saying:

> **q: q is false — (*i.e.*, q is 0.0 true) — or q is 0.*a* true, or q is 0.[*a*+*b*] true or q is 0.[*a*+*b*+*c*] true, but q is not 1.0 true.**

To put it in symbolic notation:

$$\mathbf{q} \equiv \sim\mathbf{q} \vee (0.a)\mathbf{q} \vee (0.[a+b])\,\mathbf{q} \vee (0.[a+b+c])\mathbf{q}$$

or …

$$\mathbf{q} \equiv (\sim\mathbf{q} \vee (0.a)\mathbf{q} \vee (0.[a+b])\,\mathbf{q} \vee (0.[a+b+c])\mathbf{q}) \wedge \sim\mathbf{q}$$

A paradox results, since upon opening out the above terms we get:

$$(\mathbf{q} \equiv \sim\mathbf{q} \wedge \sim\mathbf{q}) \vee (\mathbf{q} \equiv ((0.a)\mathbf{q} \wedge \sim\mathbf{q}) \vee (\mathbf{q} \equiv ((0.[a+b])\mathbf{q} \wedge \sim\mathbf{q}) \vee$$
$$(\mathbf{q} \equiv ((0.[a+b+c])\mathbf{q}) \wedge \sim\mathbf{q})$$

It many be noted that *all* the terms separated by ∨ ("or") operators are disallowed by the axioms and rules of inference of the above-defined five-valued logic. And since in five-valued logic no *other* terms are allowed, a paradox does result.

Or, in plain language,

(1) If **q** has a truth-value of 1.0 (*i.e.*, **q** is totally or absolutely true), then **q** is neither 0.0 true nor 0.*a* true nor 0.[*a*+*b*] true nor 0.[*a*+*b*+*c*] true. A contradiction results.

(2) If **q** is 0.0 true (*i.e.*, **q** is totally or absolutely false) then by inference it asserts an absolute truth (*i.e.*, **q** is 1.0 true). Again, a contradiction results.

(3) If **q** is 0.*a* true, then too by inference it asserts an absolute truth (*i.e.*, **q** is 1.0 true). Once again, a contradiction results.

(4) If **q** is 0.[*a*+*b*] true, then too by inference it asserts an absolute truth (*i.e.*, **q** is 1.0 true). Once again, a contradiction results.

(5) If **q** is 0.[*a*+*b*+*c*] true, then too by inference it asserts an absolute truth (*i.e.*, **q** is 1.0 true). Once again, a contradiction results.

Since there are *no* other possibilities, a paradox results due to the above five contradictions.

The same sort of result may be obtained for ***n***-valued logic, if ***n*** is any finite integer greater than 2. (*Note:* there cannot be a *one*-valued logic, let alone a *zero*-valued logic!)

HOWEVER:

4. Assume a Logic where the Number of Truth Values is Unbounded:

Let a proposition **q** admit of truth-values whose total number is *unbounded* — which is to say, the number of truth-values the proposition **q** can admit of is not limited to any *pre*-determined number ***n***.

This means that if there is a pre-determined number ***n***, any proposition **p** may bear truth-values as follows:

(1) 100% (or 1.0) true, which is to say totally or absolutely true,

(2) 0.999…9 (to ***n*** decimal places) true,

(3) 0.999…8 (to ***n*** decimal places) true

(4) …

$(10^n$-1)	0.000…1 (again to ***n*** decimal places) true,
(10^n)	0.000…099…9 (now to ***n+1*** decimal places) true,
$(10^n$+1)	0.000…099…8 (again to ***n+1*** decimal places) true,
$(10^n$+2)	…
$(10^{n+m}$-1)	0.000…000…1 (to ***n+m-1*** decimal places) true, and
(10^{n+m})	0.0 true (*i.e.*, totally or absolutely false.)

In this respect, the following definition applies:

> **To say of a proposition p that it is "0.*x* true", where *x* is any finite integer greater than zero, means that it neither 100% (or 1.0) true nor 0% (or 0.0) true — *viz.*, false, but somewhere in-between: its exact position in between the values 1.0 and 0.0 being exactly 0.*x* (whichever integer *x* may be); and as a result, its degree of membership in the set of all propositions that are true is 0.*x*.**

Thus:

> **To say of a proposition p that it is 0.*x* true means that it belongs to the set of all propositions that are totally true by a degree of 0.*x*, and to the set of all propositions that are totally false by a degree of 0.(1-*x*).**

Now bearing in mind the pre-determined number ***n***, assert the following proposition **q**:

> **q: q is 0.0 true (i.e., totally false) or q is 0.000…1 [worked out to *n* decimal places] true or q is 0.000…2 [also worked out to *n* decimal places] or … q is 0.999…9 [once again worked out to *n* decimal places] true.**

This is the same as saying:

> **q: q is 0.0 true (i.e., totally false) or q is 0.000…1 [worked out to *n* decimal places] true or q is 0.000…2 [also worked out to *n* decimal places] or … q is 0.999…9 [once again worked out to *n* decimal places] true, but q is not 1.0 (or totally) true.**

Or, in symbolic notation:

$$\mathbf{q \equiv (0.0)q \vee (0.000\ldots1_{\mathit{n}})q \vee (0.000\ldots2_{\mathit{n}})q \vee \ldots (0.999\ldots9_{\mathit{n}})q}$$

or …

$$\mathbf{q} \equiv ((0.0)\mathbf{q} \vee (0.000\ldots1_n)\mathbf{q} \vee (0.000\ldots2_n)\mathbf{q} \vee \ldots (0.999\ldots9_n)\mathbf{q}) \wedge \sim\mathbf{q}$$

[Here, the notation $\mathbf{0.}\boldsymbol{uvw...y_n}$ — where ***u, v, w, y*** and ***n*** are each of them any digit between 0 and 9 inclusive — means the term $\mathbf{0.}\boldsymbol{uvw...y}$ is worked out to ***n*** decimal places.]

Now:

(1) If **q** is 1.0 true (*i.e.*, totally or absolutely true), then **q** is neither 0.000…1 true nor 0.000…2 true nor … 0.999…9 true. If so, q is *not* 1.0 true. A contradiction results.

(2) If **q** is 0.000…1 true, then by inferece **q** is also 1.0 true: which, according to the (final) assertion of **q** itself, it is not. A contradiction results.

(3) If **q** is 0.000…2 true, then by inference again, **q** is also 1.0 true. Again, a contradiction results.

(4) If **q** is 0.000…3 true, then by inference once again **q** is also 1.0 true. Once again, a contradiction results.

(5) …

(10^n) If **q** is 0.999…9 true, then once again **q** is also 1.0 true. Once again, a contradiction results, though just barely.

BUT note that here, the proposition q *can* take on *yet another* truth-value, one that is *not* on the above list! Thus for example:

(10^n+1) If **q** is — say — 0.000…01 true (worked out to ***n+1*** decimal places), then **q** is not *totally* or *absolutely* false (*i.e.*, **q** is not 0.0 true), but then, neither is it *totally* or *absolutely* true (*i.e.*, **q** is not 1.0 true.) No contradiction results, and thus no paradox.

In standard symbolic notation modified for logic in which truth-values can be unbounded, this becomes:

$$\mathbf{q} \equiv (0.0)\mathbf{q} \vee (0.000\ldots1_n)\mathbf{q} \vee (0.000\ldots2_n)\mathbf{q} \vee \ldots (0.999\ldots9_n)\mathbf{q} \wedge (0.000\ldots01_{n+1})\mathbf{q}$$

Since there can always be a truth value of **q** worked out to ***n*** decimal places, where both ***n*** and ***m*** are finite integers greater than zero, the truth value of **q** can be anything between 1.0 and 0.0 *exclusive* of 1.0 and 0.0, *and* worked out to (***n+m***) decimal places!

Thus, for example, if *x* is any finite integer greater than zero,

(10^n+*x*) **q** is $\mathbf{0.}\boldsymbol{uvw\ldots yz}$ true (where ***u, v, w, y*** and ***z*** are each of them any digit between 0 and 9 inclusive, and the whole expression $\mathbf{0.}\boldsymbol{uvw\ldots yz}$ is worked out to

n+1 decimal places), in which case **q** can be *either* or *neither* 0.0 true (i.e., false) *or* or *nor* 0.000…1 [worked out to ***n*** decimal places] true *or* or *nor* 0.000…2 [also worked out to ***n*** decimal places] *or* or *nor*… 0.999…9 [once again worked out to ***n*** decimal places] true, and *yet* be *neither* 1.0 true (*i.e.,* absolutely true) *nor* 0.0 true (*i.e.,* absolutely false)!

Thus no contradiction ensues; and thus, again, no paradox results.

A Simple Contradiction is Not a Paradox

It should be noted that a simple contradiction — or even a long series of contradictions — does not *by itself* constitute a paradox. A paradox *only* results if, given any particular set of parameters, nothing *but* contradictions result. (It will have been noticed that this has been indicated on pages 6 and 7 above: under three-valued logic, for example, if there are only two outcomes, a paradox does *not* result.)

Thus it is necessary to show that within the given parameters, *all* possible outcomes to the problem being considered *do* result in contradictions. And as a consequence, the number of outcomes to that problem must not only be *denumerable,* but also *bounded* by a given (not just a *finite*, but a *given*) number: a number, in other words, which equals the number of outcomes possible within the given parameters. (Since the parameters are given, so too must this number be: for example, two-valued logic must have two outcomes, three-valued logic, three outcomes, … **n**-valued logic, **n** outcomes.)

If *all* the outcomes are not exhausted, there *might* be an outcome which does *not* result in a contradiction — in which case there would *not* be a paradox! After all, a paradox can only be validly called a paradox if it can be *established* that it is one: namely, by examining each and every possible outcome, and showing that they *all* result in contradictions, without a *single* exception.

The above three paragraphs, in a nutshell, constitute the crux of the argument on which this Essay is based.

Some Objections Anticipated and Refuted

(1) It may be objected that if **q** has a truth-value that is *not* on the list that follows the terms “**q: q is**”… , namely:

> **0.0 true (i.e., false), or**
> **0.000…1 [worked out to *n* decimal places] true, or**
> **0.000…2 [also worked out to *n* decimal places] true, or**
> **… or**
> **q is 0.999…9 [once again worked out to *n* decimal places] true**

… then **q** must be *totally* false, and cannot even be the least bit true. This is not the case *under a system of logic that admits of an **unbounded** number of truth-values,* because under a system of logic in which **q** can take on a number of truth-values that is unbounded (*i.e.*, not limited to a pre-determined number ***n***), then the proposition **q** can have a truth-value that is *not* zero *nor* 1.0, and *yet* satisfy the requirement that it has a truth-value that is not on the list, the total number of whose members is limited to the number ***n***.

Thus for example if **q** is ($\mathbf{0.000\ldots05}_{n+1}$) true, then under such a system of logic it is considered to be exactly halfway between (**0.0**) true and ($\mathbf{0.000\ldots1}_{n}$) true, but not *totally* false.

And if **q** is ($\mathbf{0.000\ldots000\ldots01}_{n+m}$) true, where ***m*** is a *very, very* large integer, then under such a system of logic it is considered to be *very, very* close to (**0.0**) true and yet not *totally* false.

(2) It may be objected that the list may not be limited to the number ***n***, but may be limited to a number greater than ***n***, say (***n***+***m***). This only makes is necessary to show that under a system of logic in which the proposition **q** can take on an unbounded number of truth-values, a truth-value of **q** worked out to (***n***+***m***+***o***) decimal places (where ***o*** is yet another finite integer greater than zero) would still not be on the list. As long as the number to which the list is limited is finite, the proposition **q** can take on a yet greater number of truth-values.

(3) It may be objected that the list need not be finite, and that proposition **q**, namely

> **q: q is 0.0 true (i.e., false) or q is 0.000…1 [worked out to *n* decimal places] true or q is 0.000…2 [also worked out to *n* decimal places] or … q is 0.999…9 [once again worked out to *n* decimal places] true, but q is not 1.0 true**

… can be re-written as follows:

> **q: q has any truth-value between 0.0 and 0.999…(recurring without end) inclusive of 0.0 and 0.999…(recurring without end), but q is not 1.0 true**

… in which case a paradox *would* result.

However, this argument is not valid, for it will be seen that there is no difference whatsoever between 0.999…(recurring without end) and 1.0. To establish this conclusively, we find that difference, by subtracting 0.999…(recurring without end) from 1.0 — and the answer is 0.000…(recurring without end), between which and 0 there is no difference whatsoever.[iii]

As a consequence, the re-written proposition is the same as saying:

> **q: q has any truth-value between 0.0 and 1.0 *inclusive* of 0.0 and 1.0, but q is not 1.0 true.**

However, **q** *cannot* be re-written as above without asserting the very paradox, the existence of which, under a logic admitting of an unbounded number of truth values, *remains to be proven.* The argument "begs the question", and is therefore logically invalid.

(4) It may be noted that the list *must* be *either* limited to a finite number or *not* limited to a finite number. There is no third choice. As a result, in either case the "Liar Paradox" can be avoided in a system of logic in which a proposition **q** can take on a number of truth-values that is unbounded (*i.e.,* not limited to any given pre-determined number).

Conclusion

As a result of the above arguments, it must be concluded that under a system of logic in which a proposition **q** can take on a number of truth-values that is unbounded (*i.e.,* not limited to any given predetermined number), the "Liar Paradox" can be avoided altogether.

Endnotes

[i] It can be proved, using two-valued logic, that two-valued logic cannot itself be a universally valid method of reasoning. (This was recognised by Aristotle himself, the originator of two-valued logic, who admitted — perhaps reluctantly — that there *are* meaningful sentences that can be made but which cannot be either true or false: such as the sentence "There will be a sea-battle tomorrow".)

As a result, assuming the universal validity of two-valued logic as a method of reasoning results in a paradox even more damaging to two-valued logic than is the "Liar Paradox".

The argument for demonstrating this is as follows:

1. Under two-valued logic, a statement must be either true or false — *no other choices are allowed.*

2. Under two-valued logic, therefore, something either exists or it does not exist. No other choice is allowed.

3. Now as a hypothesis, assume that free will (or, synonymously, choice) does *not* exist: that there is, in other words, no possibility of *choosing* from among a number of different courses of action.

4. If free will (or choice) does not exist, then a person cannot possibly *choose* to believe one belief and reject another.

5. Thus if one person believes that free will (or choice) does *not* exist, whereas another believes that it *does*, they could never come to their respective conclusions by any sort of argument or reasoning. They must each believe what they believe simply because neither of them can have any choice in the matter.

6. As a consequence, it would be impossible to tell which of them is right.

7. And as a corollary, it would be impossible to know whether the belief that free will (or choice) does not exist is really *true*.

8. Strictly under two valued logic, if it is *not* possible to know of any belief that it is true, then it *must* be possible to know of its opposite (or, synonymously, of its negation) that it *is* true.

9. The opposite (or negation) of the belief that free will (or choice) does *not* exist is that free will (or choice) *does* exist.

10. Therefore, and again strictly under two-valued logic, it *must* be true to say that free will (or choice) *does* exist, and as a corollary, that it cannot *not* exist — which in turn proves conclusively that the assumption made earlier in No. 3 above must be false.

11. Given now that under two-valued logic, free will *must* exist, now it must also be acknowledged that any statement made about the future which entails the exercise of free will (or choice) *must* be neither true nor false, for how the future will actually turn out will depend on how the free will or choice will be exercised.

12. Thus it is rigorously proven that under strictly-applied two-valued logic, it *must* be possible to make statements that can be neither true nor false: which contradicts No. 1 above.

13. Consequently two-valued logic cannot be a universally valid method of reasoning.

14. *Q.E.D.*

[ii] It should be noted that in two-valued propositional logic, *every* conclusion can be derived from either the operators {"~" and "∨"} or {"~" and "∧"} (*i.e.*, {"not" and "or"} or {"not" and "and"}) *exclusively*. Thus the operator "≡" (*i.e.*, "materially equivalent to") can be derived from them too. One consequence of this is that the above reasoning constitutes a kind of "proof" of the Liar Paradox, although if the operator "≡" is included in the list of symbols, the Liar Paradox cannot, strictly speaking, be *proved* in two-valued symbolic logic, but is taken as an (unproved) axiom.

[iii] It may be argued that there *is* an infinitesimally small difference, greater however than zero, between 0.999 … (recurring without end) and 1.0. However, if that is admitted, then it must also be admitted that the proposition **q** can thereupon bear a truth-value that is infinitesimally even smaller and yet not zero. In either case, the paradox is removed.