ESSAY:

ON GEOMETRY

(With Particular Reference to the Theory of Relativity)

by

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Started Thursday, December 13, 2001
Finalised February 4, 2002

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IMPORTANCE OF GEOMETRY

Geometry is perhaps next only to logic and (numerical/algebraic) mathematics as being one of the most important tools enabling us to understand and manipulate the material world. Pretty much all of technology, ancient as well as modern, depends on geometry in one way or another for its development, and even for an understanding of the scientific basis behind it. Indeed without geometry we may say that there would be no science or technology at all!

EXACTLY WHAT *IS* GEOMETRY?

However, it is not all that clear exactly what geometry *is.* Buckminster Fuller, one of the most brilliant engineers of the 20th century, and inventor of — among other things — the Geodesic Dome (than which hardly anything else can be called more geometrical!) often denied the very *existence* of geometry in what he called "the real world". And he had a point. Or rather, he argued that he *didn't* have a point, and neither did anyone else: that, in other words, a geometrical point — defined as an entity having a position but no dimension — did not even exist in the *real* (read: *material*) world. And this is obviously true.

And if points do not exist in the material world, neither do lines, triangles, squares, circles or for that matter Geodesic Domes. The only entities that can possibly exist in the material world are *approximations* to these geometrical elements and figures.

But of course this does not prevent these geometrical elements and figures existing purely in the *mind,* as abstractions. One may *define* a point as an entity that has only a position but no dimension — and even if *nothing* on earth fits that description, the definition *itself* is still valid.

(It's a bit like a unicorn, if "a unicorn" is *defined* as "a silvery horse-like animal with a horn growing out of its forehead". Although there ain't no such animal, the *definition* itself is still valid, in that if there *were* such an animal, it *would* be a unicorn as above defined.)

LIMITATIONS OF DEFINITIONS

It may be thought that if what's been said above is true — and it certainly seems to be so — then one may define anything any way one chooses, especially if the thing being defined is a mere mental or imaginary entity or construction. Now this may be true in the case of *single* definitions, especially if after the entity has been defined, no further *use* of its definition is made; but it is most emphatically *not* true in the case of multiple definitions, or even of single definitions of which *use* is intended to be made, especially in a real- (read again: *material-*) world context.

That's because logically and practically speaking a definition cannot be nonsensical, nor go counter to the common usage of language, nor contradict any other definition with which it is being used, nor be incompatible with clear observation.

As an example, one may *define* oneself a billionaire, and leave it at that — and that, *by itself,* would be fine; but if one tries to make *use* of such a definition in the real world order to buy oneself one's dream house or the latest and greatest sports car, one may well run counter to the clear observation that one doesn't actually *have* a billion dollars.

Likewise, although one may define a point as an imaginary entity having a position but no dimension, one *cannot* define it as having one or more *dimensions* but no *position.* That would be a nonsensical statement, even for a wholly imaginary entity, and therefore not a definition at all. Nor can one define a point (in the singular) as having *two or more* positions simultaneously. That too would be nonsense, and as a result, would also not fit the definition of "definition".

Nor, if one is using the English language, may one define "and" as a noun, or "if" as a pronoun. Such definitions would run counter to the use of the English language. (One exception: these days one can define almost anything as a *verb!*) ☺

DEFINITIONS IMPLICIT IN OTHER DEFINITIONS

Moreover, in most definitions, there are other definitions that are implicit. For example, the reason why it is nonsensical to define a point as having one or more *dimensions* but no *position,* is that if we are using the English language (as it is commonly understood), then the definition of "position" is implicit in the definition of "dimension". Anything possessing *any* dimensions at all must have at least *two* positions, and of course can have more than two. It is impossible for something having dimensions not to have *any* position at all — and although some philosophers (and of course many politicians) do assert that their position is that they *have* no position, such a position is rather obviously self-contradictory.

Now all the above may seem quite obvious, and the reader may wonder why I am actually saying all these things; but bear with me: I shall use these statements to critically examine and eventually refute the logical validity of a large part of what is commonly understood to be "geometry" — especially much of what passes for non-Euclidean geometry, as well as the so-called "geometry" used in the Theory of Relativity.

DEFINITIONS WHICH CONTRADICT OTHER DEFINITIONS

Besides, of course, geometry one has to start off with not just one, but several, definitions; and if using these definitions one intends subsequently to *prove theorems,* then obviously no definition may contradict any other: because if they did, then one would be able to obtain self-contradictory theorems from them, and that too would be nonsensical and illogical. If a statement — and a mathematical, geometrical or logical theorem, when put into words, is neither more nor less than a statement — completely and absolutely contradicts another, then *both* of them cannot possibly be true, because if they were, it would also be true that both of them *could* be true, and thus both statements could be true and not true at the same time: and this would make a mockery of the very notion of truth.

And of course the whole aim of logic, science, *etc.* is to find out what's true and what isn't;[1] and this goal cannot be attained if the very concept of truth is jettisoned. (If it *were* jettisoned, then anything and everything could be true, and it *would* be possible for you to buy your gorgeous mansion in Ottawa's prestigious Rockcliffe Park neighbourhood as well as your Bugatti *Veyron* for just ten Canadian dollars each.)

LIMITATIONS OF DEFINITIONS

It should also be borne in mind that one cannot possibly define *everything.* At some stage in the definition process the words used in the most *basic* definitions have to be taken as undefined — or in other words, they have to be understood *intuitively:* for otherwise one would have to define *every* word in terms of *other* words, and since in every language there are only a finite number of words, this would eventually lead to a logical circle.

[1] It is often maintained that logic *per se* has nothing to do with the truth — that it only deals with symbols and their correct (or incorrect) manipulation. But those who make such an assertion refer to *symbolic* logic alone in saying this. And they do not realise that symbolic logic itself could never come into existence without a superior logic which is *not* symbolic, and whose processes of reasoning have perforce to be *understood.* After all, logically speaking, and in the final analysis, the very test of *any* statement — including the statements made by those who deny any connection of logic with the truth — is whether the statement is *true* or not, and not merely whether it correctly manipulates symbols or not! (See also the section "Logic *vs.* Symbolic Logic" further on in this very Essay.)

As a result, *any* definition of a given term must also adequately *fit* the intuitively-understood meaning of that term; for if it didn't, then the terms which perforce have to be left undefined and have to be intuitively understood would *not* be intuitively understood.

(This may also seem all too obvious, but it has been equally obviously been forgotten in the attempt to generate, and assert as logically valid, all kinds of non-intuitive "geometries".)

LOGIC AS THE BASIS OF ALL MATHEMATICS (INCLUDING GEOMETRY)

Perhaps at this stage it is as well also to be quite clear what geometry *isn't.* More specifically, one must agree that it is definitely *not* a system of thought whose elements can ever run contrary to *logic:* an illogical geometry is a very contradiction in terms. There may be some controversy whether all of mathematics — if we use that term as including geometry — can be *derived from* logic, as is claimed by some and denied by others; but there can be absolutely *no* controversy that neither mathematics (*i.e.,* the numerical and algebraic parts of it) nor geometry can ever be *illogical.*

Thus it may be pertinent to clearly understand what *logic* is, in this context. Of course we are speaking here only of binary or Aristotelian logic, because this is the only kind which is used for all of mathematics (including geometry). A theorem — *viz.,* a logical conclusion arrived at by a clear process of reasoning — when we are speaking of mathematics and/or geometry, is either correct or it is not: it cannot be *almost* correct, nor can it be *both* correct *and* incorrect … nor can it be *neither* correct *nor* incorrect.

LOGIC *VS.* SYMBOLIC LOGIC

Those who claim that mathematics can be derived from logic also say that it can be derived from *symbolic* logic. Their argument is simple enough. Symbolic logic is essentially a system of assigning symbols to all the propositions and operators of logic, and then manipulating the symbols according to a certain set of rules. This can be done in a totally mechanical fashion: that is, *without actually understanding what is being done.* The same thing can be done with mathematics — indeed that is how calculators, which understand nothing, are able to "do" mathematics. Since symbolic logic and mathematics are so similar in this respect, it is relatively easy to simply add the symbols and operators which pertain to mathematics, and the rules of their manipulation, to those which pertain to symbolic logic, and thereupon obtain a system which includes both.

However, it is not to be forgotten that a system of symbolic logic cannot *itself* be constructed without a process of reasoning which is properly and adequately *understood.* (This is similar to the way a calculator cannot "do mathematics" at all without a *program* created for that purpose; and this program *has* to be created by an intelligence which actually *understands* what that purpose is intended to be.)

Thus it is clear that pure, unadulterated or *non*-symbolic logic must *precede* symbolic logic, and thus must precede mathematics as well (including geometry.) And there is no better confirmation of this fact than the very existence of the disciplines of *meta*logic and *meta*mathematics, which cannot possibly be developed without an actual *understanding* of the concepts involved.

Having noted that such an understanding *must* exist at the basis of *all* logic and mathematics, including geometry, we are forced to conclude that in their very fundamental basis, every term used in all of these disciplines must carry some *meaning.* No doubt the meaning which each of the terms carries may either be intuitively understood or explicitly defined; but unless the meaning is actually *understood,* the entire study of logic, mathematics and geometry loses all value, at least as regards the pursuit of truth. (And the pursuit of truth is of paramount importance in *every* discipline — for if *any* statement, regardless of the discipline in which it is made, is not true, why should it be believed at all?)

DEFINITIONS NEEDED FOR GEOMETRY — EUCLID'S DEFINITIONS

So now let us tackle the concepts needed specifically for proving the theorems of geometry. It is obvious that a few such are *absolutely* indispensable, and may be called "basic concepts of geometry". Among them are surely the concepts of: "point", "line", "straight line", "curved line", "angle", "right angle", and "circle". Others like "triangle", "square", "polygon", "sphere", "cube", *etc.* may perhaps be defined in terms of these basic concepts. But without the basic concepts, geometry as we know it cannot even exist.

Euclid in his *Elements* (see <http://aleph0.clarku.edu/~djoyce/java/elements/toc.html>) has already defined pretty much all the concepts of geometry, but some of them are rather unsatisfactorily defined from a modern point of view. Most importantly, his definition of "straight line" is much too hazy. We note that he has defined "a straight line" as "a line which lies evenly with the points on itself." But if we use one of the more commonly-held meanings of the word "evenly", then it could validly be said that *all* lines lie evenly with the points on themselves. There is no way, using such a hazy definition, to say *precisely* what a *curved* line is in contrast to a *straight* line.

And we *need* the definition of a *straight* line if we want to be clear as to what a *dimension* is. Euclid has not defined "dimension" anywhere in his *Elements.* No bloody wonder, then, that others, taking advantage of Euclid's oversight in this respect, have attempted to come up with "geometries" possessing any number of dimensions, regardless of whether such a thing could possibly exist or not, even in the mind or the imagination, let alone in the material world!

DEFINITIONS NEEDED FOR GEOMETRY — OUR PRELIMINARY DEFINITIONS

Let us try then, as a preliminary exercise, to define the concepts we need for *modern* geometry as follows — accepting of course the notion that we may need to refine our definitions as we proceed to find lacunae in our preliminary definitions, just as we have found in Euclid's:

1. *Point* : an (imaginary) entity possessing a (single) position but no dimension.

2. *Line* : the (imaginary) path traced out by a point moving through space.

3. *Straight line* : the shortest distance between any two points.

4. *Curved line* : any line that is not straight.

5. *Angle* : when two straight lines intersect, four angles are obtained at and around the point of intersection.

6. *Right angle* : four right angles are formed when two straight lines intersect in such a manner that all four of the angles thereby formed are congruent with each other.

7. *Circle* : the path traced out by a point which is moving through space in such a manner that it is always equidistant from another (single) point.

From these we can obtain such things as *triangle* (when three straight lines intersect with each other, a triangle is formed between the three points of intersection).

POSITION MUST BE IMMOVABLE

Now — *and this cannot be emphasised enough!* — the very first of these definitions deals with the concept of "position". It is clear, of course, that position is a *relative* concept: a position is only one which is relative to some *other* position. No position exists *by itself.* No space can have just *one* position!

So there must be *some* position in our space which we can *start* with. It can be chosen quite arbitrarily, of course, but this arbitrarily chosen position *must* exist to begin with.

In simpler words, we must have a "here" before we can have a "there". And we must in addition assume that "here" is *immovable.* If "here" were to move, then "there" would move too, and then there would be points moving all over the imagined space in which geometry unfolds. Then no two points would be able to define any *given* straight line, for the line itself would change from moment to moment!

It is obvious that Euclid has forgotten to mention this requirement, taking it to be "understood". But these days, with the Theory of Relativity rearing its ugly head, we have to bear in mind that the requirement of immovability of at least *one* position which can be called "here" — or in Cartesian terms, at co-ordinates 0,0,0 — must come *first,* before dealing with *any* of the rest of geometry.

CRUCIAL IMPORTANCE OF A "STRAIGHT LINE" IN GEOMETRY

It is also to be noted (and this too cannot be emphasised enough) that the concept — whether explicitly defined or intuitively understood — of a *straight* line, as opposed to a line that is *not* straight, is *absolutely crucial* to *any* type of geometry. (Obviously a "triangle" made up of squiggly lines is no triangle at all, and certainly can't be used to prove any of the theorems which deal with *genuine* triangles!)

Admittedly the above definition of "straight line" is not *altogether* satisfactory; but in order for there to be geometry of *any* nature there has to be *some* definition of "straight line" which *(a)* fits the intuitively-understood meaning of that term and *(b)* also enables us to clearly differentiate a straight line from one that isn't. So we shall start off with the above definition as a preliminary exercise, and if necessary introduce others as we go along to see whether they are any better.

Before we go on to the definitions of "square", "polygon", "sphere", "cube", *etc.,* however, we need to define "dimension". This is all the more so because of the appearance of the word "dimension" in our very first definition: the definition of "point".

DEFINITION OF "DIMENSION"

Defining "dimension" is however not easy. As a preliminary exercise perhaps we may define "dimension" in the following three stages:

8. A space in which only one straight line can exist — in the imagination, of course, because it does not exist in the material world anyway! — is defined as possessing *one* dimension;

9. A space in which two straight lines intersecting at right angles to one another can exist — and again, when we say "exist", we mean "exist in the imagination" only — is defined as possessing *two* dimensions, and

10. A space in which three straight lines, all intersecting at a *single* point at right angles to each other, can exist — and once again, this can be in the imagination only — is defined as possessing *three* dimensions.

Whether it is possible to have a space in which *four or more* straight lines, *all* intersecting at right angles to each other at a single point, can exist at all, we shall examine in greater depth later; but as a preliminary consideration, we can at least say that if such a condition cannot even be *imagined,* it cannot fit the definitions given earlier, because right from our very earliest definitions we have clearly stated that the point, the line, the angle, *etc.* are all *imaginary* entities. That means that even if they cannot exist in the material world, they have to exist *at the very least* in the imagination.[2]

SOME PROPOSITIONS NEEDED FOR GEOMETRY

It is obvious, also, that if even we accept the above definitions, we cannot have any complete geometry unless we introduce some propositions, such as the ones introduced by Euclid (see again <http://aleph0.clarku.edu/~djoyce/java/elements/toc.html>.) For example, one has to be clear that there can be only *one* straight line between any two points. That's because it is impossible to *imagine* more than one such line. The moment we imagine *two* lines of equal length connecting two points, we can also imagine a *third* line, shorter than either of them, connecting the very same two points.

And from this it also follows that two straight lines can intersect, if they intersect at all, at only *one* point, not more than one. That's because if a straight line connecting the points **A** and **B** were to intersect another straight line connecting the points **C** and **D** both at a point **E** *and* at another point **F**, then there would be *more* than one straight line between points **E** and **F**, and this would contradict the above statement that there can be only *one* straight line between any two points.

It also follows from the above that although a *straight* line can exist in a space possessing only one dimension, a *curved* line cannot. The very *existence* of a curved line implies the existence of a space possessing at least two if not three dimensions in which that line must exist.

Likewise, although the surface defined by an angle or a triangle can exist in a space possessing only two dimensions, it is impossible for the surface of a *sphere* (defined as the surface traced out by a circle moving in such a manner that every point on the circle is always equidistant from a single point) to exist in a space possessing only *two* dimensions. Such a surface, being curved, has to exist in a space possessing *three* dimensions, and cannot possibly exist in a space possessing only one or two.

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[2] It has been argued that even if something cannot exist in the material world nor in the imagination, that by itself is not proof that it *doesn't* exist: it only proves that our *imaginations* are severely limited. (The example often given is that of God, or — for those who believe in the Trinity — at least the Holy Spirit, Who definitely cannot be observed in the material world nor adequately imagined in the mind. This, equally definitely, doesn't *prove* that God or the Holy Spirit has no *existence:* it merely proves that due to the limitations of our finite human minds, we can neither observe Him nor imagine Him.) But I think that in subjects such as geometry — as opposed to Spirituality — such an argument stretches the meaning of the word "exists" beyond its intuitively-understood meaning. If such a meaning of the word "exists" were accepted in *geometry* — as it is in Spirituality — then anything and everything would become possible, including miracles; and it could then be justifiably argued that we can square the circle, define π as exactly equal to 3.0, have a cube in only two dimensions, *etc., etc.* … thereby making an utter mockery of all geometry as we know it.

SOME COROLLARIES OF THE ABOVE

It is obvious from the above that on the surface of a sphere — or on the surface of an egg, say — there is no such thing as a straight line: *all* lines on the surface of a sphere must be curved. This is because any two points on the surface of a sphere can be connected (in the imagination at least) by a straight line which goes right *through* the sphere, and this line will obviously be shorter than any line that connects the same two points, and which *also* follows the surface of the sphere.

And if there is no such thing as a straight line on such a surface, there can be no such thing on such a surface as an angle, a triangle, or a right angle either: because these, by their very definitions, require the existence of straight lines.

"NON-EUCLIDEAN" GEOMETRIES

It is obvious from the above that the so-called "non-Euclidean" geometries of "curved spaces", particularly that developed by Riemann which deals exclusively with *positively*-curved spaces, cannot be developed using the above definitions, because in a *positively curved* space (such as the surface of a sphere) no *straight* lines can exist at all. And since the geometry of the General Theory of Relativity is a Riemannian type of curved-space geometry, it too cannot be developed using the above definitions.

So if we wish to have any geometries of curved space, or of more than three dimensions, we shall have to re-define such things as "straight line", "curved line", "right angle" and "dimension". Of these, again, the crucial concept is that of "straight line": for the very notion of a *curved* space implies that there is also such a thing as a space that is *not* curved!

STRAIGHT LINE

Thus if we want non-Euclidean geometries of *curved* spaces, the concept of a *straight* line — at least as a straight line is conceived *in contrast to* a curved line — must be made *absolutely clear*. (From the concept of a straight line we can, obviously, develop the concept of a flat surface.)

Intuitively of course we all know what a straight line is. However, if we *leave* the concept undefined, accepting only the intuitively-understood definition of "straight line", then there cannot be any such thing as an intuitively-understood straight line on — for example — the surface of a sphere, that of an egg, or that of a doughnut. Intuitively we all accept the notion that such surfaces are *curved,* and that lines on them are, as a result, also curved.

Thus if we want to logically generate any non-Euclidean geometries, we shall have to define "straight line" in terms of something else.

Normally that something else is distance. We may define a straight line as above, *viz.,* as the "shortest (imaginary) path traced out by a point moving through space between any two other points."

But now we have to be very clear as to what "shortest" means. In other words, we have to define it precisely. Or else, if we wish to leave this concept undefined, then it must mean *in our geometry* what the word "shortest" means to us *intuitively:* namely, a distance as measured by a rod, a tape or a string, with the help of which we may determine whether one distance is shorter than another.

POSTULATE OF RIGIDITY

But measuring distances implies that the objects used for measuring those distances are *rigid,* or at the very least non-stretchable. It would be ludicrous to say that a particular distance is shorter than another as measured by a string or a tape that *stretches* to accommodate itself to the distance, now wouldn't it. Nor could we confidently measure a distance using a rod that is *elastic.* So in addition to the postulates upon which Euclidean geometry is based,[3] unless we *intuitively* accept the notion of a straight line — *i.e.,* if we have to *define* "straight line" in terms of distance — it would seem that it is necessary to add a *further* postulate which clearly states that all measuring rods shall be completely *rigid,* and/or that all measuring tapes or strings be *non-stretchable.* Without such a postulate it seems impossible to clearly state what a straight line is, at least in terms of the concept of distance.

Indeed it is impossible, without an underlying concept of rigidity, even to say what a "location" or "position" is. That's because, as we already saw earlier, one cannot give the position or location even of a *point* without referring that position or location to something else! We can only say that a point is "there" if we already know where "here" is. And if we want to pin-point the position or location of "there" *precisely,* we need to be able to say exactly *how far* from "here" it is, and *in what direction.* But we cannot say *either* of the above without an underlying concept of rigidity.

So even for the concept of a point, which at least needs to have a location even though it has no dimension — and this is of course the most basic concept of all in *any* kind of geometry — we absolutely *need* an underlying concept of rigidity. And if we need it so very absolutely, we might as well come out and say so explicitly, by enunciating a postulate of rigidity which we should add to the other postulates of geometry.[4]

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[3] See again <http://aleph0.clarku.edu/~djoyce/java/elements/toc.html>.

[4] I am indebted for this argument to Michael Miller's excellent article entitled *Causality, Measurement and Space* published at <http://www.quackgrass.com/space.html#rigid>.

Besides, without a postulate of rigidity there would also be no way to define what a *circle* is. One cannot possibly draw a circle without some sort of *rigid apparatus,* like a compass, or at least without a string which *does not stretch.* And in any case, the only way to make sure a string does not stretch is to measure it against a rod which *is* rigid.

(Of course in the material world there is no such thing as a *completely* rigid rod or a *completely* non-stretchable tape or string, but that does not matter: we can always *imagine* such things, and since we are applying them to *imaginary* points and lines anyway, such imaginary rods, tapes or strings are quite adequate for our purposes. Besides, we can always *compensate* mathematically for the actual non-rigidity of existing rods and tapes when using them, if we know the *conditions* under which they alter their dimensions, and the *amounts* by which they alter their dimensions under the different conditions.)

EXAMINATION OF THE GEOMETRY OF CURVED SPACES

Now if we accept the postulate of rigidity (or at least that of non-stretchability) — and without it we obviously cannot have any points, nor can we define "straight line" in terms of distance, and neither can we have circles — we are forced to conclude that the inclusion of this postulate in geometry does *not* allow us to generate geometries of curved spaces *as they are presently being taught*. For example, we can never say of the surface of a sphere that the sum of the angles of any triangle on it is greater than 180 degrees, because there can *be no such thing as a triangle on the surface of a sphere* — and that, in turn, is because there cannot be any *straight* lines on the surface of a sphere: there can only be *curved* lines, which by definition are *not* straight. This includes the sphere's so-called "geodesics" or "great circles", which are rather obviously no less curved than an ordinary, run-of-the-mill or garden-variety circle on a *flat* surface. Indeed their very designation as "great *circles*" indicates this undisputed fact.

And a figure made up of three *curved* lines cannot legitimately be called a triangle, for if it *could* be so called, then such a figure on a *flat* surface could *also* legitimately be called a "triangle" — and that would make a mockery of pretty much all the theorems in geometry which deal with triangles, including arguably the most celebrated of them all, *viz.,* Pythagoras's Theorem.

Besides, without a postulate of rigidity one cannot even define a sphere: all "spheres" would be indistinguishable from misshapen blobs that look more or less like amoebae — and even each of those would be indistinguishable from any other such blob. This too would make a mockery of geometry as we know it.

So it is obvious that without *some* way to distinguish between lines that are straight from those that are not, all of geometry becomes meaningless. And if we do *not* accept the intuitive meanings of these terms, then the terms must be defined using some other concept. However, when using the concept of "distance" in our definitions we find that such a definition still does not allow us to rigorously construct geometries of curved space.

The question then is, whether we can distinguish between straight lines and curved ones using a definition of “straight line” which does *not* contain the concept of distance, whether explicitly or implicitly.

ATTEMPTS AT CONSTRUCTING ALTERNATIVE DEFINITIONS OF “STRAIGHT LINE”

It is my contention that such a definition is impossible. For example, suppose we try to define a “straight line” as “the path followed by an object in motion then that object is *not* acted upon by any external force.” This definition tries to make use of Newton’s First Law. But isn’t such a definition circular? Newton’s First Law sates that an object remains at rest or in uniform motion in a straight line *unless* acted upon by an external force. By saying so it assumes that we already *know* what a straight line is! If we didn’t, how would we ever know whether the object was being acted upon by an external force, or wasn’t?

In other words, what Newton’s First Law tries to do is define the concept of “force” in terms of an object moving in a straight line. It is assumed that we all *know* what a straight line is, and that we can all *imagine* an object moving uniformly in a straight line. The law essentially states that if an object deviates from the straight and uniform path, it must be *because* it is being acted upon by an external force. That way we can get a handle on the concept of “force”, which is not quite as intuitive as that of a straight line.

Likewise it seems to me ludicrous to define a straight line as a “geodesic”, as is attempted by those who believe in the General Theory of Relativity. A geodesic is by its very *definition* a line in a *curved* space, and that very fact precludes *straight* lines from fitting that description.

ANOTHER DEFINITION OF “STRAIGHT LINE”

In order to get around it, those who believe in the geometry used in Relativity — or in other geometries of curved spaces — often try to define a straight line as the “shortest distance between any two points *as measured only in a defined space*.” Thus the shortest distance between any two points *on the surface of a sphere* is a geodesic, because the defined space is “the surface of a sphere”. Thus any line that goes through a space *other* than the one defined — such as a *really* straight line that goes right *through* the sphere — is assumed to be “impossible” or “undefined within the given parameters”.

There are at least two problems, however, with such a definition. In the first place, it goes counter to common usage of the English language: much like calling a person who has a million dollars of *Monopoly* money a “millionaire”. It is highly unlikely that even *McDonald’s* will sell him so much as a hamburger or a small bag of fries for that kind of money. Or it’s like the guy said: “You can call your newfangled salmon soufflé anything you want — just don’t call it apple pie, because it ain’t.” It would be perfectly fine to call the lines on the surface of a billiard ball "geodesics", or by some other neologism; but to call them "straight lines" doesn't seem right, for they certainly aren't so in any *commonly accepted* sense of the term.

But an even stronger argument against such definitions is the fact that in order to *define* the given space itself, one needs *a priori* the concept of a *genuine* straight line, either as it is intuitively understood or as it may be specifically defined in terms of some other thing. How, otherwise, could one define the given *space* at all, if such a space is to be *curved* — such as the space on the surface of a sphere, or of an egg-shaped body, or a toroid, or indeed *any* sort of curved space whatsoever? The very *definition* of "a curved space" presupposes the notion of a *genuine* straight line, because without it, you can't even define curvature! And such a straight line cannot be obtained by the above definition, *viz.,* "the shortest distance between any two points *as measured in a defined space*", because such a definition already *presupposes* the notion of a straight line: a notion which, moreover, contradicts the above definition. So the above definition is circular, and thus illogical.

THE DEFINITION OF "DIMENSION"

There is also the problem which arises if we do not already have either an intuitively-understood notion of a genuine straight line — or a definition of "straight line" in terms of something else — namely, that we cannot obtain a clear notion (again, either intuitively understood or explicitly defined) of "dimension". For example, if *any* line were to be defined as possessing only one dimension, then how would one determine the number of dimensions required for any given line *to exist in?* Obviously — and in any case *intuitively* — a line shaped like an arc requires a space of *two* dimensions to exist in, and a line shaped like a corkscrew requires a space of at least *three* dimensions to exist in. In a one-dimensional space such lines cannot possibly exist — at least not unless the meaning one ascribes to the term "dimension" is radically different from the commonly-understood meaning of that word.

This is all the more so because if we admit the definition — as given above — that *any* line is to be defined as existing in a space possessing only *one* dimension, there could be no difference between a straight line and a curved one.

This problem exists even if we try to define "dimension" in some way which does *not* include a right angle. For example, one may define "dimension" as follows: a *point* which is moving defines *one* dimension; a *line* which is moving laterally to itself defines *two* dimensions; and a *surface* which is moving laterally to itself defines *three* dimensions. (Note that in this definition, the word "laterally" is left undefined, and is therefore intended to be intuitively understood. Thus we *cannot* say "a volume moving laterally to itself defines *four* dimensions", because the intuitive meaning of the word "laterally" does not *allow* for a volume to be moving "laterally" to itself.)

Here we do not have any such thing as a "right angle" in the definition — because the intuitive meaning of "moving laterally" does not absolutely *require* that the movement be at *right* angles to the given line or surface: *any* angle will do, as long as it isn't *no* angle at all — and so it may be thought that such a definition circumvents the requirement that we should define what a right angle is before we can define what a dimension is. But this, although true enough, is certainly not sufficient: for under this definition, we would have no way to distinguish between a straight line and a curved one — nor, in fact, a way to distinguish between lines of different degrees and types of curvature. *All* lines under the above definition are equally straight or equally curved; and that would make an utter mockery of geometry of *any* kind.

NECESSITY OF THE CONCEPTS OF BOTH "DIMENSION" AND "STRAIGHT LINE"

Thus we come to realise that for *any* kind of geometry to be logically valid, it is necessary to have *a priori* the concepts — whether intuitively understood or explicitly defined — of *both* "dimension" *and* "straight line". Just *one* of the two, *without* the other, cannot suffice. Without the concept of a "straight line", geometry becomes meaningless: there can be no such thing as a triangle, a square, or even a circle, what to speak of cubes and spheres and dodecahedrons. On the other hand, without the concept of "dimension", geometry becomes equally meaningless: one would not, under those circumstances, be able to distinguish between a point, defined as an entity having *no* dimension, from a straight line, which perforce must have *one* dimension, or from a flat plane which must have *two,* or from a volume which must have *three.*

This above requirement — and I just don't see how one can get away from it — would seem to indicate that there is no logical validity to Gaussian and Riemannian geometry. Gauss's and Riemann's geometry — or elliptical geometry as it is sometimes called — *requires* the existence of curved surfaces to *begin* with; and obviously the lines on such a surface would, by that very requirement, have to be *curved,* not *straight.* And equally obviously — indeed by its very definition! — a *curved* line is not a *straight* one … though to read the texts expounding Riemannian geometry one would imagine that the authors couldn't care less about such a "minor" distinction.

As a matter of fact a perusal of pretty much all the text books and e-texts on the subject shows that the authors almost invariably write "line" when they actually mean "*straight* line", hoping thus to pull the wool over the eyes of the reader right from page one. It is equally clear that if the reader explicitly inserts into these texts the word "straight" before the word "line", the nonsensical nature of the text becomes all too apparent.

STUDIOUS AVOIDANCE OF THE WORD "STRAIGHT"

See for example *"Spherical geometry"* by Prof. David C. Royster of the University of North Carolina at <http://www.math.uncc.edu/~droyster/math3181/notes/hyprgeom/node5.html>. We see that he writes:

> If great circles are to be lines, then we can measure the angle between two intersecting great circles as the angle formed by the intersection of the two defining planes with the

> plane tangent to the sphere at the point of intersection. … With this definition of angle, we can form triangles on the sphere whose interior angle sum is greater than two right angles. In fact, we will show that the interior angle sum of all triangles on the sphere is greater than two right angles.

Note how studiously and carefully the good Professor avoids the use of the word "straight". Had the word "straight" been introduced in the very first line at the appropriate place, as follows, the attempt to trick the reader into accepting the utterly unacceptable would have been all too evident:

> If great circles are to be ***straight*** lines, then we can measure the angle between two intersecting great circles as the angle formed by the intersection of the two defining planes with the plane tangent to the sphere at the point of intersection. … *(etc.)*

Obviously, calling a circle — or even a *part* of a circle — a *straight* line is self-contradictory. It makes a mockery of the meanings of both the words "straight" and "circle", like calling a salmon soufflé an apple pie; it doesn't take a genius to realise rather quickly that it just ain't one.

And any attempt to claim that such "geometries" can be reflected, even approximately, in the real (read again: material) world — as proponents of General Relativity love to do — is like attempting to pay for a *real* Porsche *911* with *Monopoly* money.

And thus the rest of the arguments attempting to "prove" theorems from such a nonsensical definition are reduced to what farmers in India call cow-dung, and for which a somewhat more pejorative term is to be found in North American slang.

LOBACHEVSKY'S GEOMETRY

In contrast to Riemann, Lobachevsky seems to have had a somewhat better argument for the existence of *more* than one straight line parallel to any given straight line, in that he at least doesn't *start off* with a contradiction. (Riemannian assertions about no contradictions *arising* from the assumption that all parallel lines eventually intersect each other remind one of the assertion made by the farmer who claimed there was no animal manure in all his lands, making that assertion while standing ankle-deep in a pile of the stuff! And of course those of Lobachevsky's supporters who claim that he was speaking of *straight* lines on *curved* surfaces step into the same pile of doo-doo, or at least into a similar one.)

But Lobachevsky *himself* seems to have been smarter than that. His argument hinges on the unspoken realisation that *even* in a flat plane, no straight line can be of truly *infinite* length, but only of an *indefinite* length. (If there *were* a line — straight *or* curved — of a truly *infinite* length, the number of metres — or kilometres, or light-years, or indeed any unit of length one can think of — in it would be greater than the number of numbers, or in other words, greater than *any* number[5] … which is a clear contradiction in terms: a *number* obviously cannot be "greater than *any* number"!)[6] Since even in a flat plane the straight lines must end *somewhere,* there can be an indefinite number of straight lines going through a single point which is not on that line, and which other straight lines do *not* intersect the given straight line. And if "parallel lines" are defined as "straight lines lying on a single flat plane which do not intersect each other", then again there *could* be an indefinite number of straight lines parallel to a given straight line and going through a given point — *even* if we restrict ourselves to a flat plane. And this, as far as it goes, is true enough.

What's *not* true is that the above definition of the term "parallel" is not sufficient: *truly* parallel straight lines do not only not intersect, they don't even get closer together (nor, for that matter, do they get farther apart). If they do, then we can hardly speak of them as *parallel* lines, now can we. It would be, if not *quite* like calling a salmon soufflé an apple pie, at least calling the former a *filet de sole amandine,* or the latter a bran muffin. It just ain't so.

TOPOLOGY INCLUDED IN ORDINARY GEOMETRY

It is sometimes argued that topology, the geometry of curved two dimensional surfaces, gives results what are completely verifiable, and thus topology must be correct; and as a consequence, that the geometry of curved surfaces must also be correct. And this, by itself, is true.

But it is to be remembered that all of topology can be obtained from the geometry of three dimensional volumes. It is, in fact, a mere portion of ordinary three dimensional geometry, which is *not* being challenged here.

For instance, the geometry of the surface of a sphere is *included* in the geometry of the sphere as a whole, so it is no wonder, of course, that the geometry of the surface of a sphere is valid.

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[5] Note that the *number* of numbers cannot be infinite, because if it were, it would contradict the definition of "number" — as per Peano's axioms, for example, or the axioms of set theory as developed by Zermelo and Fraenkel, later extended by John von Neumann — according to which, for instance, if $\mathbf{x}$ is a number, then $\mathbf{x + 1}$ is *not* equal to $\mathbf{x}$. (If $\mathbf{x}$ is infinite, then obviously $\mathbf{x + 1 = x}$ … and thus $\mathbf{x}$, if it *is* infinite, cannot be a *number* as per the axioms used to define what numbers are.) — Besides, every natural number *without exception* must have a finite number of digits, and thus every natural number *without exception* must be finite. And the number of natural numbers cannot be greater than *any* natural number, for then we would have the contradiction of a natural number being greater than *any* natural number — or in other words, a natural number which both *belongs* to and does *not* belong to the set of natural numbers!

[6] From this it also follows — contrary to much of what is taught in schools and universities these days — that the number of points on a line can also not be *infinite,* only *indefinite.* If they *were* in fact infinite, we would again obtain "a number greater than any number", which is a clear contradiction in terms. (See also *A Simple Argument Against Cantor's Diagonal Procedure* at <http://homepage.mac.com/ardeshir/ArgumentAgainstCantor.pdf>.)

But from this it cannot be argued that the geometry of a curved *three* dimensional space is valid. Nor can it be argued from the above that on the surface of a sphere there are any straight lines whatsoever. To argue thus is to be slipshod in one's logic.

EUCLID'S POSTULATE OF PARALLELS

It may be noted that up till now we have made no mention of Euclid's Fifth Postulate, the Postulate of Parallels, which states: "If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, will intersect on that side on which the angles are less than two right angles." (He leaves it unsaid, of course, that this holds only for straight lines in a single flat plane — that is, in *two* dimensions only; if we apply his words to straight lines in *three* dimensions it is obvious that there is no absolute *necessity* for *any* two straight lines to intersect one another, no matter what.)

Alternatively — if he had wished to leave out the unsaid requirement that those three straight lines all lie on a single flat plane — he could have said that if one straight line intersects two *other* straight lines at right angles to them, the latter two straight lines will *never* intersect one another.

Others have tried to replace Euclid's Postulate of Parallels by an alternative postulate which defines parallel straight lines as those which are always equidistant from one another. This definition, which is also valid, does not require the definition of "dimension" either.

The reason why we have left out this Postulate, however, is that if we already *have* the definitions of "straight line" and "right angle", we can derive therefrom the definitions of "right-angled triangle" and "flat plane", and thence the definition of "rectangle": namely two congruent right-angled triangles which lie on a single flat plane and share their hypotenuse. From this definition it is easily proved that the opposite sides of a rectangle, no matter how long those sides may be, are always parallel to one another in the sense of *all* the above definitions: they never meet, are always equidistant from one another, and the angle between any two adjacent sides of a rectangle is always a right angle.

Thus although it may be true that Euclid's Fifth Postulate is not derivable from *his* other four Postulates, it is nevertheless derivable from *our* postulates; and thus we need not take it as a postulate but rather as a (provable) theorem.

ARITHMETICAL AND ALGEBRAIC INTERPRETATION OF GEOMETRY

Of course we must still explain the fact that geometry can be interpreted arithmetically and algebraically with an astonishing degree of correlation. The prime example is Pythagoras's Theorem. If a right-angled triangle has three sides of lengths $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{h}$ ($\mathbf{h}$ being the length of the hypotenuse), then the lengths $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{h}$ are related according to the algebraic formula $\mathbf{a}^2 + \mathbf{b}^2 = \mathbf{h}^2$.

And if three straight lines **a**, **b** and **c** are so connected that **a** is at right angles to **b** at one of the ends of **b**, and **c** is at right angles to **b** at its other end, such that **c** would also be at right angles to a line parallel to **a** if such a line were attached to the junction of the lines **b** and **c**, then the length of the line **h** joining the other ends of the lines **a** and **c** would be related to the lengths of **a**, **b** and **c** in the relation $\mathbf{a}^2 + \mathbf{b}^2 + \mathbf{c}^2 = \mathbf{h}^2$.

These theorems *can* be proved geometrically, so that there is no doubt that the algebraic formulae given above are correct.

But — and this is the big "but" — it seems ludicrous to "generalise" from this, and say that that if *four* or more straight lines **a**, **b**, **c**, **d** … **n** are so connected that **a** is at right angles to **b** at one of the ends of **b**, **c** is at right angles to **b** at the other end of **b**, and **d** at right angles to **c** at the other end of **c** such that **d** would also be at right angles to a line parallel to **a** and *also* be at right angles to a line parallel to **b** if such lines were attached to the junction of the lines **c** and **d**, *etc., etc.,* then the length of the line **h** joining the other ends of the lines **a** and **n** would be related to the lengths of **a**, **b**, **c**, … *etc.* in the relation $\mathbf{a}^2 + \mathbf{b}^2 + \mathbf{c}^2 + \mathbf{d}^2 + \ldots + \mathbf{n}^2 = \mathbf{h}^2$. That's because such additional straight line(s) cannot be generated! Thus such a theorem *cannot* be proven geometrically.

Indeed from the purely *geometrical* viewpoint such a theorem cannot be *dis*proved either: in pure geometry it is a nonsensical statement which can be neither true nor false — as nonsensical as Marvin Minsky's celebrated expression "Colourless green ideas sleep furiously."

(Note that we are not speaking here about a "geometry" of curved spaces — which starts off with the rather obvious contradiction of curved lines claiming to be straight — but of ordinary or *flat* spaces possessing *more* than three dimensions. Thus in such a case we don't so much find a contradiction in terms, as a set of terms which results in a *nonsensical* statement.)

"IF … THEN" ARGUMENTS

It is often claimed that it doesn't *matter* that one or more additional straight lines as above cannot actually be generated: all we are saying by the above theorem is that *if* such lines *could* be generated then the theorem above *would* be true: much like in the case of the unicorn mentioned earlier in this Essay.

But there is a logical flaw in the above argument: it also assumes — in addition to assuming that such lines could be generated — that subsequently the theorem of which the algebraic interpretation is given as $\mathbf{a}^2 + \mathbf{b}^2 + \mathbf{c}^2 + \mathbf{d}^2 + \ldots + \mathbf{n}^2 = \mathbf{h}^2$ could also be *proved* geometrically: in other words, that if we assume that such lines *could* be generated, a *proof* would exist in geometry that the algebraic formula $\mathbf{a}^2 + \mathbf{b}^2 + \mathbf{c}^2 + \mathbf{d}^2 + \ldots + \mathbf{n}^2 = \mathbf{h}^2$ is true. But with what justification can one *assume* the existence of a *proof?* Surely a *proof* is, by its very definition, something that has to be *derived logically,* not merely *assumed.* If anything is merely *assumed,* then in logic that can only be called an axiom or a postulate, not a proof.

The problem is, then, that we can *postulate* that such straight lines could exist, but that by itself would not be sufficient to provide a *proof* of the above theorem. Nor can the above formula be taken, not as a theorem, but as an additional (unproven) postulate, because then we are assuming in advance that which we intend to prove! That's not proper logic, as any sixth-grader will be able to tell.

AN INTERPRETATION OF GEOMETRY IS NOT GEOMETRY

It is also to be remembered that an *interpretation* of geometry is not *itself* geometry: it is an *interpretation* of geometry. *(Duh!).* It sounds rather redundant to say this, but it seems necessary to say it all the same, because in modern physics — or rather, what goes for "physics" these days — such interpretations are *themselves* asserted to be geometry. A formula such as the one given above, namely $\mathbf{a}^2 + \mathbf{b}^2 + \mathbf{c}^2 + \mathbf{d}^2 + \ldots + \mathbf{n}^2 = \mathbf{h}^2$, is not *itself* part of geometry, and has no proof in arithmetic or algebra *apart* from its being an interpretation of a geometrical theorem — for in pure arithmetic and algebra one can always find *some* values of **a**, **b**, **c**, **d**, … **n** and **h** for which this formula is correct, as well as *other* values for **a**, **b**, **c**, **d**, … **n** and **h** for which the formula is wrong. Thus in pure arithmetic or algebra, the formula itself is *not* a theorem — a "theorem" being defined as a formula for which a *proof* exists.

Since a proof does not exist for such a formula in pure arithmetic and/or algebra, and since it does not exist in geometry either, with what justification can one assert that it is a theorem of *mathematics* at all? Surely one cannot. And yet such formulae, or others like them (such as the formulae for calculating spatial/temporal intervals between events in Relativistic "space-time") are asserted all the time as being *theorems.* Such assertions obviously cannot be upheld as having any logical foundation.

THE MEANING OF A SET OF SYMBOLS

It has also been claimed that no contradictions arise from assuming that four or more dimensions exist, and then using "Cartesian" co-ordinates to prove theorems in such spaces. But such an argument confuses two separate meanings of the word "proof".

If one restricts oneself to symbolic logic *alone,* a "proof" in symbolic logic has, of course, *no meaning whatsoever,* because the symbols which constitute that proof have no meaning themselves; all that is being done is manipulation of a finite number of (meaningless) symbols according to a finite number of rules. This, as we said earlier, is how a computer or a calculator — which understands nothing — can "do mathematics": all it does is manipulate symbols according to certain rules, called programs.

But once these symbols are *interpreted* so as to make each of them *mean* something, then any "proof" obtained therefrom must also *mean* something: for a proof is after all a set of those very symbols, all of which are now ascribed *meanings.* This is how conscious beings like us humans "do mathematics": we *mean* something when we say we have a mathematical proof of something.

But what sort of meaning can be ascribed to a theorem which *a priori* assumes something that can have *no* meaning: *viz.,* one or more dimensions in *addition* to the three we can observe, or for that matter even imagine? Surely to think of such a "proof" as having any meaning at all stretches the meaning of "meaning" well beyond its breaking point. In such a case, the contradiction that arises is not in the theorem itself, but in tacitly assuming that a meaningless set of symbols *does* in fact have some esoteric sort of meaning — even though it is clear that the meaning cannot be grasped by the mind.

Thus, although sets of symbols purporting to be theorems of a "geometry" of four or more dimensions may well be obtained without any contradiction arising from the rules for manipulating these symbols, that by itself does not render them theorems of any *meaningful* geometry. All they can be is a set of theorems of symbolic logic — which, by the very definition of "symbolic logic", means that these theorems must be *totally devoid of all possible meaning.*

THE MEANING OF GEOMETRY

Now it is obvious that there can be no such thing as "meaningless geometry". Indeed it is questionable whether one can even have meaningless *mathematics* — or more accurately, whether a set of symbols of even *pure* mathematics (of which no practical application is possible) should not be more accurately called "pure symbolic logic". It is only when the symbols of pure symbolic logic are *interpreted* so as to derive *mathematical* theorems therefrom that we get what can accurately be called "mathematics". Surely *without* such an interpretation we are left with theorems of pure *symbolic logic* alone: *viz.,* a set of meaningless symbols which are manipulated according to certain rules. They cannot be called "mathematics" in any *adequate* sense of the term.

Thus it seems to me to stretch the meaning of the term "geometry" much too far to claim that "geometries" of four or more dimensions can have any *meaningful* existence at all. As Buckminster Fuller rightly pointed out, even a geometry of two or three dimensions cannot exist in the material world, but only in the imagination. So if we have a geometry that cannot exist *even* in the imagination, with what justification can it be called "geometry" at all? Should it not more correctly be called "symbolic logic", and a clear caveat enunciated at its very gates: *Lasciate ogni comprensione voi ch'entrate* ("Leave all notions of meaning behind, O ye who enter here")?

MEANING AND LOGIC

It should be realised that in logic — not symbolic logic but logic proper, from which symbolic logic is *derived* — what we *call* something has considerable importance. This is because, as every first-year student of logic knows, the most basic logic of all must begin with *propositions.* A proposition is, by its very definition in the context of logic, a statement that is either true or false.[7] A statement having *no meaning at all* cannot, of course, be either true or false, and is therefore *not* a proposition, and as a consequence cannot be used in logic. And an *ambiguous* statement — a statement having *more than one* meaning — is *also* not a proposition, because its truth or falsehood is *indeterminate;* or more accurately, its truth or falsehood may be determined according to the *meaning* ascribed to one or more of the word(s) or term(s) used in it. If a word or term in such a statement is ambiguous, the entire proposition can be rendered ambiguous, and thus incapable of being reliably used even in the most basic logical reasoning.

Now calling a geometry of more than three dimensions "geometry" at all would render the word "geometry" ambiguous, and therefore incapable of being reliably used in a logical argument. If we wish to use such a concept in a logical argument we ought to call it by some other name, such as "geo-*myth*-ry" (it being as mythological a "reality" as any described in *The Lord of the Rings!*)

But observe how very rapidly this changes all sorts of scientific arguments in which "geo-myth-ries" of more than three dimensions are used, such as the Theory of Relativity (both the Special and General Theories). Once we replace the word "geometry" in such arguments by the term "geo-*myth*-ry", we very clearly see that none of them can apply to the *real* (read once again: *material*) world. This includes, of course, the "geometry" of Relativity.

THE GEOMETRY OF MINKOWSKI "SPACE-TIME"

It is also to be noted that the so-called "geometry" of Minkowski space-time, developed for the express purpose of establishing Einstein's arguments of Special Relativity on a firm geometrical and mathematical footing, must be included in the category of "geo-*myth*-ry". It is of course true enough that a point moving through space in any given direction describes a straight line — *i.e.,* a *one*-dimensional continuum; a straight line moving through space in any given direction at an angle to itself describes a flat plane — *i.e.,* a *two*-dimensional continuum; and a flat plane moving through space in any given direction at an angle to itself describes a volume — *i.e.,* a *three*-dimensional continuum. But from this it cannot be "generalised" that a volume moving through space in any given direction at an angle to itself describes a *four*-dimensional continuum, because we all know that a volume moving through space in *any* direction merely describes a still larger volume, and *not a four-dimensional continuum at all!*

Similarly, although it is true that one can plot the movement of a point through a *one*-dimensional space as a *two*-dimensional graph, with time represented on the vertical axis and space on the horizontal axis, and one can even plot the movement of a point through a *two*-dimensional space as a *three* dimensional graph, with time represented on the vertical axis and space on the

[7] Admittedly this applies only to binary or Aristotelian logic, but since all of mathematics — including Geometry — is derived using binary logic alone, we shall restrict ourselves to this sort of logic here.

two horizontal axes, it makes no sense to say that one can plot the movement of a point through a *three*-dimensional space as a *four*-dimensional graph, with time represented on the vertical axis and space on the horizontal axes, because we need *three* horizontal axes to plot the three dimensions of space, and one just cannot *have* three horizontal axes.

And if we wish to make use of the vertical axis to represent the third dimension of space, there is no axis *left* on which time can be represented.

MINKOWSKI "SPACE-TIME" INCOMPATIBLE WITH THE CONCEPT OF MOTION

It is evident also that if time is treated the same as any other spatial dimension — as is done in Minkowski "space-time", the geometrical basis of the Special Theory of Relativity — then the concept of *motion* cannot possibly exist. After all, the motion of a point is defined as a *change* of the point's spatial position *over time* — and if time is treated like any other spatial dimension, then such a concept loses all meaning.

To get over this problem, motion is treated in Special Relativity as a *relative* concept: a body is said to be in motion or not depending *exclusively* on the relative motion of the one who observes it. If the observer is moving along with the body being observed, at the same speed and in the same direction, *that* body would be motionless to *that* observer, even though it may be *in* motion as seen by any *other* observer. This is called the "the Principle of Relativity".

However, there is a problem here, in that the Theory of Relativity does not stop at the *Principle* of Relativity: it goes on to postulate that the speed of light is a constant for *all* observers: thereby asserting that as far as light waves (or photons) are concerned, motion is *not* a relative concept but an *absolute* one. This is obviously contradictory to the Principle of Relativity — as Einstein himself realised only too well (see his 1920 book *Relativity: The Special and General Theories,* at <http://www.bartleby.com/173/7.html>).

RELATIVITY OF SIMULTANEITY

In a valiant attempt to overcome this rather obvious contradiction, Einstein devised his famous "Train" thought-experiment, the object of which was to prove that two events separated by a distance which are simultaneous for one observer are definitely *not* simultaneous for another observer moving relative to the first observer at a velocity **v**. In other words, Einstein tried to prove that simultaneity of events separated by a distance is relative, and depends on the speed of the observer.

For those who are unfamiliar with Einstein's "Train" thought-experiment, we shall describe it hereunder. Suppose — argued Einstein — that a train is moving in a straight line past a railway platform at a uniform speed **v**. Suppose a man is standing on the platform; and suppose a passenger is standing in the train exactly at its mid-point. Suppose that as the train rolls by the platform, two lightning bolts strike the two ends of the train, leaving burn marks on the front end of the engine and rear end caboose, and also on the track below them. (Einstein himself does not speak of the burn marks, but we shall introduce them so as to be certain of the locations where the lightning bolts struck.) Suppose the man on the platform *observes* the two lightning flashes simultaneously. Suppose that he afterwards measures the distance from where he was standing to the burn marks left by the lightning on the tracks, and discovers that he was standing exactly mid-way between these two burn marks. Since by the postulate of the constancy of the speed of light this means that the flashes must have taken identical amounts of time to reach his eyes from the moment they occurred, the two lightning bolts must have *struck* the train simultaneously.

Now, asks Einstein, would the passenger *also* see the two flashes simultaneously, or not? He argues that she would *not,* because the train — along with any and all of its passengers — is *moving* at a speed **v** *towards* the light waves (or if you prefer, photons) coming from the engine end, and *away* at speed **v** *from* the light waves coming from the caboose end.[8]

EINSTEIN'S "TRAIN" THOUGHT-EXPERIMENT CONTRADICTS THE POSTULATE OF THE CONSTANCY OF THE SPEED OF LIGHT

Einstein forgets, though, to bear in mind that if the above argument is correct, then according to the passenger on the train, light should also take less time to go *all the way* from the front end of the train to its rear end than it does to go all the way from the rear end of the train to its front end! In other words, the speed of light, as far as the passenger is concerned, should *not* be a constant, but should be dependent on the motion of the train. This conclusion contradicts Einstein's other postulate, the postulate of the constancy of the speed of light for all inertial observers.

And it can hardly be argued that as far as the *passenger* is concerned, the distance from rear end of the train to its front end is greater than the distance from the front end of the train to its rear end! The *distance* the light travels in *both* directions must be the *same,* at least in the frame of the passenger: that distance being, of course, *the length of the train.* And if light travels the *same* distance in *different* amounts of time, the speed of light *cannot* be a constant.

Thus the argument proffered by Einstein to try to prove, with his "Train" thought-experiment, that simultaneity is relative, *contradicts* the postulate of the constancy of the speed of light for all inertial observers. At least one of the two must be wrong. In other words, either the argument that simultaneity must be relative is wrong, or the postulate of the constancy of the speed of light is wrong — or both are wrong. But they both cannot be *correct.*

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[8] In other words, Einstein is tacitly asserting that as observed by the passenger, the speed of the light waves (or photons) coming to her from the *front* end of the train is **c+v** — where **c** is of course the speed of light — and the speed of the light waves (or photons) coming to her from the *rear* end of the train is **c-v**. This obviously contradicts the postulate of the constancy of the speed of light. But since the assumption is tacit, it is missed by those who are not careful enough to spot it.

Note that according to Einstein's Principle of Relativity, from the point of view of the *passenger* on the train, the train is *not moving at all,* whether towards or away from the two spots where the flashes of lightning struck the train — these spots being, in *her* frame, the burn marks left by the lightning strikes on the engine and the caboose. And since the passenger is, by Einstein's other hypothesis, standing exactly at the *mid-point* of these two burn marks on the train, and since by the postulate of the constancy of the speed of light, the speed of light must be just as constant for her as for the man on the platform, then if as concluded above the two lightning bolts struck the train simultaneously, the *passenger* on the train must see the two resulting flashes simultaneously as well.[9]

In other words, all *three* of Einstein's assertions cannot possibly be correct. Either the Principle of Relativity — according to which *all* motion is relative — is wrong; or the postulate of the constancy of the speed of light for all observers, regardless of their motion relative to the source of the light, is wrong; or the assertion that simultaneity is relative is wrong.

As we shall see below, in fact, it turns out that logically speaking, *all three* must be wrong.

IMPOSSIBILITY OF THE POSTULATE OF THE CONSTANCY OF THE SPEED OF LIGHT TO BE CORRECT

In any case, the postulate of the constancy of the speed of light cannot possibly be correct. It is to be noted that in the light of the Principle of Relativity, an alternative way of expressing the postulate of the constancy of the speed of light is to say that the *relative* speed between any photon — or, if you prefer, any light wave — and *anything* else must always be equal to **c**. However, if this were truly the case, it would be *impossible* for any *two* photons — or light waves — to leave a source of light simultaneously and also arrive at the eyes of an observer simultaneously: for if they did so, then the *relative* speed between them, as determined by that observer, would have had to have been be *zero:* and this would contradict the above conclusion that according to the postulate of the constancy of the speed of light, their relative speed must be neither more nor less than **c**.[10]

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[9] Note that it cannot be asserted that the two lightning bolts struck the train simultaneously *only as far as the man on the platform is concerned,* but not as far as the passenger on the train is concerned. Such an argument would be assuming in advance the very conclusion, for the truth of which Einstein is attempting *via* this thought-experiment to establish a proof.

[10] It may be objected that the Theory of Relativity does not allow for a measurement to be made of the relative speed between two *photons,* since no observer can be travelling at the speed of light. But we need only accompany one of the photons with an observer travelling, not quite at the speed of the photon itself, but at a speed which is so close to the speed of the photon as the difference not capable of being measured within the margin of error of the available instruments. This is allowable according to the Theory of Relativity, and yet also satisfies the requirements of the argument above.

Alternatively, if any two photons were to move off in exactly *opposite* directions simultaneously from a single source — as would be the case for multiple pairs of photons if a flashbulb were to flash extremely briefly — then after a time **t** (as measured by an observer stationary relative to the source) each of these two photons will have moved away a distance **d = ct** away from the source (also as measured by an observer stationary relative to the source). But since the two photons are moving away in exactly *opposite* directions, the distance between them after a time **t** will be equal to **2d = 2ct**. Since the two photons will have travelled, according to the above-mentioned observer, a distance of **2d = 2ct** in a time **t**, the relative speed between them, as determined by this same observer, will be **2d/t = 2ct/t = 2c** — which contradicts the postulate of the constancy of the speed of light, according to which the relative speed between a photon and anything else must always be **c**.

(It may be remembered that *even* if the so-called Relativistic "length contraction" and "time dilation" do exist, the *relative velocity* of an object with respect to another must nevertheless still observer-independent. That is because according to the postulate of the speed of light, the velocity of the speed of light must *itself* be observer-independent. And *any* relative velocity whatsoever can always be expressed as a percentage — or if you will, a fraction, or ratio — of the speed of light. So if the speed of light is to be observer-independent, then so must any fraction or ratio of it be.)

"TRICKS" PLAYED BY GEOMETRY

Now let us come to the various "tricks" played on human minds by Euclidean geometry, because of the fact that Euclid, when devising his definitions, common notions, postulates and propositions for geometry as he understood it, did not think about cases when his imaginary entities *and* the observer were *both* moving relative to the imaginary "space" in which the geometry was supposed to be unfolding (or *in* which the geometric figures of his geometry were supposed to be drawn). That is to say, Euclid — and pretty much everyone after him — has/have tacitly assumed that there is a "space" *in* which geometry unfolds; and the observer, the person thinking about the geometrical theorems which he is expounding, is always *stationary* with respect to this "space". *Within* that "space", imaginary entities like points, lines, planes and even volumes may themselves move, but the *observer is always supposed to be stationary relative to that undefined "space".* (We already discussed this when speaking of the Immovability of Position.)

But no one thinks about what happens when *both,* the imaginary entities which are the *subject* of geometry, *and* the observer who is thinking about them, *move* relative to that space.

One simple example of such a case, with which most of us are familiar, is when rain which is falling *vertically* downwards relative to a horizontal road is observed by us when we are in a car *moving* at speed along that very road. We *see* the rain falling, not *vertically* downward, but at a *slant* to the road. This phenomenon is called aberration.

Indeed if we have a set-square bolted firmly on to the fender of the car, so that one bar of the set-square is vertical to the road and the other bar is trailing horizontally in the direction of the car's movement, we will actually observe, by comparing the falling rain with the vertical and horizontal bars of the set-square, that the rain *is* in fact falling, not *along* the vertical bar, but at a *slant* to it: thereby "proving" to our satisfaction that observation confirms that the rain is *not* falling vertically.

But if a similar set-square were placed on the *road itself,* stationary relative to the *road,* then by looking at the rain falling along *that* set-square, we would observe that the rain *is* in fact falling *vertically!* So which is the *real* "reality" — *is* the rain falling vertically or *isn't* it?

"REALITY" IN THE CONTEXT OF GEOMETRY

The question as posed above seems to have a fairly clear answer on the Earthly plane: we would normally answer that the rain *is in reality* falling *vertically* to the road, but as observed from a moving car it merely *appears* to be falling at slanted angle to the road. But from the point of view of *physics* this answer is not very satisfactory, for if it were, we would be able to tell that the car is in motion, and the Earth (with the road) is at rest. But nowadays we *know* that the Earth is *not* at rest at all, but is moving through space. Not only is it rotating around its own axis, but is also revolving around the centre of mass of the Solar system, which in turn is revolving around the centre of mass of the Galaxy. And perhaps the Galaxy too is in motion in the Universe on some as-yet unknown trajectory. Indeed if we are to be scrupulously honest, we have to say that we have *no idea* exactly in *what* trajectory the Earth is moving! All we can say, along with Galileo, is *"Eppur si muove".*

There is no way, however, to define what a *right angle* is under these circumstances. Indeed if we imagine the road to be in *outer space,* with the rain replaced by, say, cosmic ray particles all falling at right angles to the road, they would either hit the road at right angles, or they would not, depending on whether the road too was imagined to be moving or not!

And it is to be noted that although the above example deals with rain or cosmic ray particles, which are in the *material* realm, the *same* result would be obtained if instead of rain or cosmic ray particles we were to *imagine* point-like "particles" falling in the same direction as the rain (or the cosmic "rain", as the case may be). In other words, this is a *geometrical* phenomenon, *entirely in the imagination,* and not a *material* or *physical* one at all.

STRAIGHT *VS.* CURVED PATHS

This becomes even more clear when we realise that, what to speak of the difference between a right angle and any other kind of angle, even the difference between a *straight* path and a *curve* disappears when there are *two* moving objects — imaginary or otherwise — and one of them is moving uniformly along a straight path while the other is *accelerating* along a straight path. If we observe a point which is moving uniformly and rectilinearly while we ourselves are *accelerating* along a straight path at right angles to the trajectory of the movement of the point, the trajectory would be *curved* to us, even though by very definition the point is moving along a *straight* path! (This is the essence of Einstein's famous "Elevator" thought-experiment, which relies for its illusory effect on precisely this trick of geometry.)

Thus if we wish to preserve any sense of *genuine* geometry at all, we have to specify that the "things" (points, planes, volumes or whatever) may be considered to be moving in the space in which we imagine that geometry to be unfolding, but the *observer* or *geometer* should not be considered to be moving *as well.* We cannot assume that *we,* the *geometers* (*i.e.,* the people *doing* the geometry) are moving also. If we do so assume, then the concept of right angle and straight line loses all meaning, and thereby all the theorems of geometry, including Pythagoras's Theorem, become meaningless. Indeed, as we pointed out earlier, even the notion of "location" or "position" becomes meaningless.

Thus if we want to preserve geometry as we know it, we have to add a postulate to geometry which says, in effect, that for geometry to be valid, only the imaginary entities which are the *subject* of geometry may be considered to be moving through the space in which we imagine our geometry to be unfolding; but the "observer" of these entities — the geometer, that is to say the person *doing* the geometry in his imagination — must be considered to be stationary at all times. The very validity of the concept of "position" or "location", as well as that of a right angle or of a straight line, depends on this postulate — and *ipso facto,* the validity of most of the theorems of geometry depends on it as well.

EINSTEIN'S "ELEVATOR" THOUGHT-EXPERIMENT

In Einstein's "Elevator" thought experiment we find this postulate violated, and as a result we can say conclusively that it does *not* represent a genuine geometrical theorem. Those who are familiar with the Theory of Relativity will of course be familiar with this thought-experiment also; but for the benefit of those who are not, I shall outline the relevant part of it hereunder. Consider an elevator with a man in it, says Einstein, located in empty space far from any detectable gravitational field, which is made to accelerate in some way (for example, with the help of a rocket — Einstein doesn't mention rockets, but we know that rockets would indeed do the needful). Suppose the acceleration of the elevator is at a *constant rate in a straight line* past a ray of light which is being propagated in that very same empty space at right angles to the line of acceleration of the elevator. Then if that ray of light were admitted into the elevator through a small hole in one of its walls, it would *curve downwards* — as observed by the man in the elevator — and as a result would hit the opposite wall a little *lower* than the spot *directly* across the elevator from the small hole.

But note that this is the case *only as observed by the man in the elevator.* It is the *man in the elevator* who is taken to be the "observer" here. But since he is constantly *accelerating,* his velocity is constantly *changing,* and thus he *cannot* be assumed to be stationary at *any* point in time! And by our above considerations, it is impossible for geometry to be valid when the observer *himself* is moving in the very space in which the geometry is imagined to be unfolding.

And this is proved by the fact that, as judged by another person who is observing — or imagining — the elevator as if from the *outside*, and who as a result is *not* moving along with the elevator in the space in which the geometry is imagined to be unfolding, the ray of light would be always travelling along a *straight* path! (For an illustration of this in the form of easy-to-understand pictures, see <http://www.hopefoundationint.com/pages/principle.html>.)

Thus for a *genuine* geometrical theorem — that is to say, for a geometrical theorem to be *logically valid* — we absolutely *need* to have the observer stationary at all times. If the observer is moving *in* the very space in which the geometry is imagined to be unfolding, and if in addition there are other imaginary (or even non-imaginary) entities which are moving through that very same space, then when considering the paths or trajectories of those entities, geometry and its theorems become logically invalid, in that there is no longer any meaning to a *straight* path (or trajectory) as opposed to a *curved* one. Or more accurately, a path or trajectory becomes straight or curved *depending on the state of motion* — or the lack of it — of the observer.

THE "ELEVATOR" THOUGHT-EXPERIMENT WITH A TWIST

And no better demonstration of this fact can there be, than a consideration of what the man in the elevator would observe were the elevator to be imparted, not a *constant* acceleration, but an alternating *acceleration and deceleration in different directions* (say by means of rockets mounted on its roof as well as under its floor — and perhaps on its front and its rear as well — all firing alternately and at different times.) Under such circumstances, the man in the elevator — were he to be strapped, for instance, to a chair in it, so that he wouldn't be flung about all over the place as a result of the different accelerations and decelerations — such a man would observe the path of a ray of light which is admitted, as mentioned earlier, through a small hole in one of the elevator's walls, to be altogether *squiggly,* and thus neither curved *merely* downwards nor curved *merely* upwards! And of course, the path of the ray would *not* appear to him to be straight at all.

Indeed by judicious firing of the rockets, the man in the elevator could be made to "observe" the path of the ray of light to be almost any shape one pleases, including a corkscrew shape.

And yet we know that the *same* ray of light, when observed (or imagined) by an observer who is — or is imagined to be — *outside* the elevator, an observer who is *not* imparted an acceleration, to such an observer the path of the ray of light would be perfectly *straight.*

So we can say with a considerable degree of confidence that the *real* path of that ray of light is as straight as an arrow, and not squiggly at all. The squiggly path as observed by the man in the elevator is a mere *illusion,* created by causing the elevator — and as a result, the man in it — to undergo acceleration and deceleration in all sorts of different directions.[11]

THE "PRINCIPLE OF RELATIVITY"

One cannot argue against the above, by the way, by saying that motion is always *relative,* and an object is neither moving nor non-moving unless it is in reference to — or *relative* to — some other object. (This, as we mentioned earlier, is known as the "Principle of Relativity".) That's because if an object is *accelerating,* then its state of motion is constantly *changing,* and as a result, if *must* be in motion at all times *relative to itself:* that is, relative to its *own* state of motion or the lack thereof at *other* times!

And the proof is, that if at any given moment in time its own motion were matched by some *other* object which was moving *uniformly and rectilinearly*, then at *that* moment the first object would no doubt be at rest relative to that second object, but that would *not* be the case at *any moment in time before or since.*

Thus the Principle of Relativity does *not* apply to accelerating objects. Only *uniform rectilinear motion* can be considered to be relative, if at all; every *other* kind of motion has got to be *absolute.*

Indeed when we think of the Principle of Relativity in greater detail, we are forced to conclude that in *some* way this Principle just *cannot* be valid. For according to this Principle, if two objects, which we may call **P** and **Q**, are moving rectilinearly and uniformly relative to one another at a velocity **v**, then according to the Principle of Relativity it should be just as correct to say that **P** is moving at a velocity **v** relative to **Q**, as to say that **Q** is moving at a velocity **v** relative to **P**. If the Principle of Relativity is correct, *either* statement is *equally* true, and there is absolutely *no* difference between them.

Now so far so good. But — and this is a very *big* "BUT" — if after a certain period of moving rectilinearly and uniformly at a velocity **v** relative to one another a force is applied to the object **P** over a period of time **t**, thereby changing the velocity of **P** to a velocity $\mathbf{u} \neq \mathbf{v}$ relative to **Q**, then if the Principle of Relativity is correct, at the *same* time the object **Q** must *also* change *its* velocity relative to **P**, namely from **v** to **u** ! In other words, the Principle of Relativity asserts that just because **P**'s velocity changes relative to **Q** over a period of time **t**, **Q**'s velocity relative to **P** must *also* change over that *same* period of time **t**, even though *absolutely no force has been applied to the object* **Q** *at all.*

[11] I am indebted for this argument to Dr Christoph von Mettenheim, in whose book *Popper Versus Einstein* (written in English but published in Tübingen, Germany by Mohr) it was, to my knowledge, first mentioned.

For remember that according to the Principle of Relativity, the *only* kind of velocity that can possibly exist is *relative* velocity: there can be *absolutely NO other kind!* So whenever we speak of "velocity" we can mean *relative* velocity *only.*

And if this is so, the Principle of Relativity must assert that a change in a body's velocity can be brought about *without* any force whatsoever applied to that body — which is quite contrary to the laws of physics.

THE THEORY OF RELATIVITY AS A PURELY GEOMETRICAL EXERCISE

For note also that although we have spoken of "objects" here above, if Einstein's argument is to be correct, we would have to speak about *totally imaginary geometrical entities.* In other words, we can only speak validly of pure geometry here, which deals entirely with a *mental* reality, but not of physics, which deals entirely with a *material* reality. It *is* possible to change the velocity of an imaginary entity without a force being applied to it; but it is *not* possible to change the velocity of a *material* object without a force being applied to it.

It can be seen from all the above arguments that the Theory of Relativity — as opposed, say, to Newton's Laws — is a *purely geometrical exercise:* in that its conclusions, if they are at all to be held as being correct, must apply only to imaginary points, lines, *etc.* and not to physical entities like you and me and cars, and planets, suns and stars.

In Newton's *Principia Mathematica Philosophiae Naturalis* the distinction is clearly made between physics and mathematics: the state of motion of a body *cannot* be altered unless the body is acted upon by an external force. But in Relativity no such distinction is made: and that is why it just *cannot* be a theory of physics.

Aside from the above-mentioned conundrum which arises from assuming that the Principle of Relativity is correct, Einstein's elevator, which was mentioned earlier, for example, should accelerate in a straight line *indefinitely* without any force or energy input. (If that were not implied, then Einstein's alleged "Principle of Equivalence" — *i.e.,* the presumed equivalence between inertial acceleration and gravitational acceleration — would not be correct: for to keep the man in the elevator experiencing a force of 1-g *inside* the elevator when the elevator is being accelerated in empty space far from any detectable gravitational field, an *input of energy* is required, and such an input must, in the *physical* world, always have *a beginning and an end;* whereas no such energy input is required to enable the man stand firmly on the floor of the very same elevator were it to be stationary on Earth: and the man — or his descendants — could experience a force of 1-g in *such* an elevator indefinitely.)

Such considerations prove that the Theory of Relativity stands or falls by its *geometrical* validity: for it has no *physical* validity whatsoever.

However, as we have seen above, there can also be no *geometrical* validity to a geometry when the observer is himself assumed to be moving in the very space in which the geometry is imagined to be unfolding. Nor can there be any validity to a "geometry" based on a postulate — the postulate of the constancy of the speed of light for all observers — which cannot possibly be correct.

THE THEORY OF RELATIVITY FALLS FLAT WHEN PHYSICAL PROPERTIES ARE INTRODUCED INTO IT

And nothing illustrates this fact better than the introduction into the Theory of Relativity of *genuine* physical properties like mass or force: whereupon Relativity falls flat on its face. Aside from the above example of the elevator, another illustration of this is the following: according to the Theory of Relativity, an object which is *accelerating* — *i.e.,* gradually moving *faster and faster* — should be *increasing* in mass as time goes by, while an object which is *decelerating* — *i.e.,* gradually moving *slower and slower* — should be *decreasing* in mass as time goes by. But, and this is a *really* big "BUT", the Theory of Relativity *also* insists that acceleration and deceleration are *equivalent to one another in every way:* or in other words, that there is *no way* to tell of an object whose velocity is *changing,* whether it is changing in the direction of an *increase* in velocity or a *decrease* in velocity! Indeed according to Relativity, any *particular* object can be both accelerating and decelerating *at the same time* — that is to say, *accelerating* relative to a *second* object, and *decelerating* relative to a *third.*[12] So if an object's velocity is at all *changing,* according to the Theory of Relativity its mass could be both increasing *and* decreasing *simultaneously* … which is of course quite impossible.

After all, mass is not a *relative* property at all, dependent on the velocity of the observer, but an *inherent* (and therefore *absolute*) property of every physical object. And the proof is that every object generates a gravitational effect on every other body, and this gravitational effect is *proportional to its mass;* and it is *not* dependent at all on the velocity of the observer, but only on the square of the *distance* between the first object and any other object on which the gravitational effect is being exerted.

[12] For instance, suppose we are in a spaceship far from any detectable gravitational field, and an enemy spaceship is retreating from us in a straight line at a uniform velocity **v**, and suppose we fire a missile at that enemy ship to try and destroy it; then from the moment of the missile's firing until it matches the enemy ship's velocity, the missile will be simultaneously *accelerating* relative to *our* ship and at the same time *decelerating* relative to the *enemy* ship.

ABSOLUTE REST

The notion that absolute rest does *not* exist is therefore rooted in the notion — which is perhaps acceptable in *pure* geometry,[13] but not in any *physical* interpretation thereof — that acceleration and deceleration are equivalent to one another in *every* way. This however cannot be the case from a *physical* perspective, because *acceleration* inevitably and *without fail* requires an *input* of energy, while *deceleration* can *release* energy. Of course deceleration can *also* be achieved by an input in energy — for example, one can decelerate a car with the help of rockets attached to the front of the car, which fire *forwards* instead of rearwards. But one can also decelerate a car conventionally, that is with the help of brakes, which heat up as a result (and which heat can be used in some other way, say to warm up a cup of coffee.) But it is impossible to *accelerate* a car and also *derive* energy therefrom for any other purpose, because acceleration *absolutely and without fail* requires an *input* of energy.

We can also reach this conclusion if we consider a closed system with a finite number of objects in it. If energy is *input into* the system, the objects will *increase* their velocities *relative to what their own velocities were* before the energy was input into the system; while if energy is *extracted from* the system, the objects will *decrease* in velocity relative to what their own velocities were before the energy was extracted. However, the increase in velocity when energy is input into the system, and the decrease in velocity when energy is extracted from the system, *need not be relative to the other objects in the system!* If the energy is input or extracted carefully enough, the objects in that closed system can *all* be moving at the *same* velocity relative to each other, both before *and* after the change in energy of the system.

THREE DIFFERENT KINDS OF MOVEMENT

From the above considerations we reach the conclusion that there must be at least *three* different kinds of movement, just as there are three different kinds of mass (*viz.,* gravitational, inertial and centrifugal, all of which are different from one another in subtle but definite ways.)

Regarding movement, therefore, firstly there is the movement of an object relative to another. It is obvious that this sort of movement cannot be an *inherent* property of an object at any time, because the same object — call it **P** — can be moving at a velocity **v** relative to another object **Q**, and simultaneously moving at a velocity **u** relative to *yet* another object **R**. Indeed the object **P** can have an *unlimited* number of velocities relative to an unlimited number of other objects *simultaneously.*

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[13] We say "*perhaps*" because as we noted above, in pure geometry there must be a "here" as opposed to any "there", and the "here" must be immovable. As a result, even in pure geometry there is at least a *tendency* to oppose the notion that absolute rest does *not* exist.

But this kind of movement cannot be the *only* kind, for as we illustrated above, it results in the absurdity of concluding that an object can *change* its velocity relative to another — that is to say, either accelerate or decelerate — *without* any force being applied to it, simply because that *other* object, *relative* to which the first object is moving, has changed *its* velocity. As a result, even Relativity at least *implicitly* accepts the notion that there is *another* kind of velocity, and that is the velocity an object has *relative to its own velocity at another (specified) time.* For even Relativity accepts the notion that a *change* in an object's velocity is absolute, and not relative.

Now *this* kind of movement *is* an inherent property of the object concerned, at least at any specified instant in time. That's rather obviously the case, because there is clearly *no other object involved!* Besides, the amount of energy required to change that object's velocity by a given amount over a given time period is *independent* of who the observer is. (If that were not the case, then as we pointed out above, the object's mass too would have to be relative, and dependent on the observer; but its mass *cannot* be relative, and dependent on the observer, because its mass generates a gravitational effect which is *not* dependent on the observer.)

And thirdly, there is the velocity an object has relative to some absolute state of rest. That's because acceleration and deceleration *cannot* be equivalent to one another, since the former absolutely *requires* an *input* of energy, while it is possible to *extract* energy from the latter.

THE PRINCIPLE OF RELATIVITY *DEMANDS* THE EXISTENCE OF A STATE OF ABSOLUTE REST

Besides, if the universe is finite, then the Principle of Relativity *itself* demands the existence of a state of absolute rest in it. That's because every finite collection of objects, no matter *how* they are distributed, *must* have a centre of mass; and in any *closed* system, the state of motion — or the lack thereof — of that centre of mass is unaffected by the movements of the individual objects which make up that system. And what more closed a system can there be than the *entire* universe (when "the universe" is defined as "everything material that exists")? Thus the state of motion — or the lack thereof — of the centre of mass of the entire universe *cannot* be affected by how its individual components move.

But then the question arises: just *what* is the state of motion of the centre of mass of the entire universe? If we accept the Principle of Relativity, that centre of mass *cannot be in motion at all,* for if the universe is defined as *everything* material that exists, there can't be *anything* else relative to which it can be in motion!

Thus the Principle of Relativity, coupled with the hypothesis that the universe is finite, absolutely *requires* that the centre of mass of the universe be at *absolute* rest. (And by doing so, the Principle of Relativity proves that it is itself, logically speaking, self-contradictory.)

LOGICAL IMPOSSIBILITY OF THE EXISTENCE OF AN INFINITE UNIVERSE

By the way, one cannot get away from this conclusion by positing an *infinite* universe. A *truly* infinite universe cannot logically exist, because if it did, it would contain more objects than there are natural numbers — *i.e.,* counting numbers — to count them! After all — and as we already pointed out earlier when discussing the number of points on a line — natural numbers *cannot* be infinite. Besides the fact that a hypothetical infinite natural number would not fit the definition of "number" — in that if that infinite natural number were designated as **n**, then **[n+1] = n**, which contradicts the axiom that for every number **x**, **[x+1] ≠ x** — it should also be borne in mind that *every* natural number *without exception* contains a *finite* number of digits, and thus *every* natural number, no matter how great, *must* be finite. So if there is a greater number of objects in the universe than there are natural numbers, we would end up with the contradiction that the number of objects in the universe is greater than *any* number. (After all, a number cannot be "greater than any number", because if it could be, then it would *both* belong to *and* not belong to the set of numbers!)

WHY ABSOLUTE REST HAS NOT YET BEEN DISCOVERED

One criticism that may perhaps validly be raised against the notion of absolute rest is, that if such a thing exists, then how is it that we have not yet found it? The answer lies perhaps in our ignorance: more specifically, in the fact that we do not know in detail the mass and motion of every object in the entire universe. If this *were* known to us, we would obviously also know just where the centre of mass of the universe lies — and that would tell us how each object in the universe is moving relative to that centre of mass.

And the proof is that it we hypothesise ourselves living in a much *smaller* universe than ours — say the size of our own Galaxy — then we conceivably *would* be in a position to know it all, and thereupon to determine its centre of mass … and *ipso facto,* of the movement of every object relative to it.

Thus to those who say — and this sort of statement is often heard — "There is no evidence anywhere in our universe for anything in a state of absolute rest", we can validly reply: "Absence of evidence is not evidence of absence!" For logically speaking, such a state *must* exist.

THE IMPORTANCE OF NOT FALLING INTO TRAPS

All the above references to the Theory of Relativity have been given here for the purpose of illustrating the ways in which one can fall into traps in geometry unless one is *very* careful. It should be obvious from the above arguments that the Theory of Relativity is a purely geometrical exercise — and one which, moreover, cannot have any logical validity. And the basic reason for this is the fact that it takes advantage of the way some things in geometry become very tricky, especially when motion is introduced into it … and even more especially, when the *observer* *himself* is assumed to be in motion. It is highly important, for the sake of logic, that one not fall into these traps.

The way one can avoid these traps is, I think, by judicious and careful use of one's reason. If one finds that a certain word — such as "motion" or "velocity" — is being used ambiguously, one should try and find different words for the two or more different meanings … or else at least make sure that in any particular case, the meaning with which the word is used is abundantly clear. Otherwise one can easily fall into the trap of making utterly incorrect statements such as "Velocity is always relative, and since a body's relative velocity can change as a result of a force applied to *another* body relative to which the first-mentioned body is moving, the latter's velocity can change even though no force whatsoever is being applied to it."

A SET OF POSTULATES AND DEFINITIONS FOR *MODERN* GEOMETRY

In light of the above, we may enunciate a set of postulates and definitions which might form the basis of *modern* geometry, since Euclid's own are seen to be lacking. Actually, not *all* of his are lacking, only some of them; and thus we need only change a few of his, add a few of our own, as follows. In this respect, the additional postulates given hereunder are, I think *absolutely crucial:* indeed Euclid *implicitly* accepts them. But it is only by being *clear* about them that we can avoid the traps into which geometry has fallen, especially with reference to non-Euclidean "geometries" and the Theory of Relativity.

Besides, it is as well to declare right from the outset that geometry and its component entities can exist only in the imagination, and not in the material world — the only things that exist in the material world can be *approximations* to the imaginary entities of geometry.

Without enunciating such a declaration, one can also get into all kinds of logically invalid statements, such as "a volume moving laterally to itself defines four dimensions."

So without further ado, let us enunciate our declaration, and our amended postulates and definitions, as hereunder:

Declaration

1. *Declaration of the Imaginary Nature of Geometry:* Geometry exists solely in the imagination, and not in the material world.

Postulates

1. *Postulate of Immobility of the Geometer:* The geometer — the person imagining the geometry to be unfolding — is at all times to be imagined as being *immobile* in the space in which the geometry is imagined to be unfolding.

2. *Postulate of Rigidity:* The space in which the geometry is imagined to be unfolding must consist of *measurable distances,* and the *only* way distances can accurately be measured is by having at least one if not more *rigid* rods with which different distances can be compared.

Definitions

1. *Point* : an imaginary entity possessing a single position but no dimension.

2. *Line* : the imaginary path traced out by a point moving through the space in which geometry is imagined to be unfolding.

3. *Straight line* : the shortest possible distance between any two points.

4. *Curved line* : any line that is not straight.

5. *Angle* : when two straight lines intersect, four angles are obtained at and around the point of intersection.

6. *Right angle* : four right angles are formed when two straight lines intersect in such a manner that all four of the angles formed thereby are congruent with each other.

7. *Circle* : the path traced out by a point which is moving through space in such a manner that it is always equidistant from another single point.

8. A space in which only one straight line can exist in the imagination is defined as possessing *one* dimension.

9. A space in which two straight lines intersecting at right angles to one another can exist in the imagination is defined as possessing *two* dimensions.

10. A space in which three straight lines, all intersecting at a *single* point at right angles to each other, can exist in the imagination is defined as possessing *three* dimensions.

CONCLUSION

From all the foregoing, the following definite conclusions may surely be drawn:

2. Geometry is of great importance in most of human endeavour.

3. Geometry in its pure form exists only in the imagination, not in the material world; in the material world only *approximations* to geometry can exist.

4. Logic is the basis of all mathematics including geometry: for no mathematics or geometry may ever be illogical.

5. No axiom, postulate, definition, *etc.* of geometry or mathematics may ever contradict another, nor may it ever contradict any conclusion reached with the help of other axiom, postulate, definition, *etc.* — for otherwise geometry becomes illogical.

6. Euclid's definitions are not completely satisfactory for *modern* geometry, for they do not allow us to define a "straight line" and a "right angle" adequately.

7. It is absolutely necessary for the very existence of the concept of "location" or "position" — and *ipso facto* for valid geometry — that the geometer must be considered to be *immobile* in the space wherein the geometry is imagined to be unfolding.

8. A series of definitions satisfactory for modern geometry is absolutely needed for it; and of these, the definition of "straight line", "right angle" and "dimension" are crucial.

9. In order to attain any satisfactory definition of "straight line", it is absolutely necessary that a "postulate of rigidity" be added to the other axioms, postulates, definitions, *etc.* of geometry.

10. When a postulate of rigidity is introduced into geometry, no logically valid "non-Euclidean" geometries are possible.

11. Arithmetical and algebraic interpretations of geometry are not *themselves* geometry, and thus the mere existence of algebraic and arithmetical formulae which correspond to certain geometrical theorems does not permit one to generalise therefrom, and thereby to claim that non-Euclidean geometries can exist despite the fact that such geometries cannot be imagined.

12. There can be no such thing as "meaningless geometry".

13. The "geometry" of Special Relativity — sometimes also called the "geometry of Minkowski space-time" — is based on a postulate which contradicts at least one theorem proved with the help of the other postulates, *etc.* of geometry, namely the Galilean theorem of addition of velocities; and thus the "geometry" of Special Relativity is not logically valid.

14. The Special Theory of Relativity is moreover founded on a logically invalid argument, namely the "Train" thought-experiment first enunciated by Einstein: because that argument tacitly assumes that the speed of light is *not* a constant for all inertial observers, which contradicts the Relativistic postulate that the speed of light *is* a constant for all inertial observers.

15. There are also other reasons that the postulate of the constancy of the speed of light for all inertial observers cannot logically be valid: for example, if it *were* valid, no two photons (or light waves) could simultaneously leave a source of light together, and also arrive together at any given destination.

16. Geometrical theorems become invalid when the "observer" (more accurately, the geometer, *i.e.,* the person imagining the geometry in his imagination) is *also* in motion — or is imagined to be in motion — in the very space in which the geometry is imagined to be unfolding.

17. In Einstein's "Elevator" thought-experiment, which is the basis of the General Theory of Relativity, the "observer" is imagined to be moving; and thus its conclusion — that rays of light would *actually* (and not merely *apparently*) "bend" when an observer is accelerating at right angles to the path of the rays — is invalid form a geometrical perspective … and this fact alone is sufficient to disprove the General Theory of Relativity.

18. It is impossible for the "Principle of Relativity" to be correct, for if it were, acceleration and deceleration would be equivalent to one another in *every* way, and thus a condition which absolutely requires an *input* of energy would be equivalent in every way to a condition which allows the *extraction* of energy … which is of course impossible.

19. The Theory of Relativity is a purely *geometrical* exercise — and a faulty one, at that — but cannot be a theory of *physics,* for it collapses when physical properties such as mass or energy are introduced into it: for according to Relativity, an object increases in mass when it is accelerated and decreases in mass when decelerated, but according to the same theory, there is *no* difference whatsoever between acceleration and deceleration … so that there should be no difference, according to the Theory of Relativity, between a body *increasing* in mass and *decreasing* in mass. (Here the Theory of Relativity fails even the "five-year-old test" — *viz.,* whether or not a five-year-old would laugh uproariously at such a claim!)

20. There are obviously three different kinds of movement, just as there are obviously three different kinds of mass; and it is wrong to use more than one kind in any single argument.

21. It is important not to fall into the traps which geometry allows us to fall into; but this will almost inevitably happen unless we define its terms and postulates adequately to begin with.

22. A declaration, and a set of postulates and definitions which might be adequate for *modern* geometry is enunciated at the end of this Essay, as being more satisfactory than Euclid's. This more modern set may, however, not be completely satisfactory either; but it is the best I myself can come up with. (It will of course be up to future generations of thinkers to refine them still further.)

COMMENTS

Comments, if any, will be most welcome. The author appreciates e-mail sent to him either at his e-mail address or postal address, both of which are given on the Title Page.